Nonlinear signal analysis: An intuititive introduction without formulae

BENESCO Lecture Series

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Nonlinear signal analysis: An intuititive introduction without formulae

Literature

H Kantz, T Schreiber, Nonlinear time series analysis, 2nd ed. Cambridge University Press, Cambridge, 2004

&

RG Andrzejak, Nonlinear time series analysis in a nutshell. Osorio et al. (eds.) Epilepsy: The Intersection of Neurosciences, Biology, Mathematics, Engineering and Physics. PDF available at www.upf.edu/web/ntsa/

Deterministic versus stochastic dynamics

A dynamical system is called deterministic if its temporal evolution is fully determined by its initial conditions. It can be defined, for example, by a set of differential equations.

For stochastic dynamics the temporal evolution is not unambiguously determined by their initial conditions. Instead the temporal evolution is governed by some random process.

Motivation

SIMULATION

- Visual inspection of exemplary signals
  - Classification as random or regular
  - Can we make this classification using the power spectrum?

Measuring the harmonic oscillator

SIMULATION

- Harmonic oscillator: a simple dynamical system
  - Temporal evolution of components
  - Signal
  - State space representation

Lorenz dynamics

SIMULATION

- Lorenz dynamics
  - Three components
  - State space representation

Reconstructing the state space

SIMULATION

- Harmonic oscillator
  - original signals and state space
  - delayed signals and reconstructed state space

Delay coordinates

SIMULATION

- We apply delay coordinates to Lorenz dynamics
  - influence of time delay τ

Trajectories of deterministic dynamics ...

SIMULATION

- Inspection of trajectories of deterministic dynamics
- Nearby traces of the trajectories

. . . do not intersect

For any given initial condition the future evolution of a deterministic dynamical system is fully determined.

$\Rightarrow$ Trajectories of deterministic dynamical systems cannot intersect. Nearby trajectory segments are aligned.

For any given initial condition the future evolution of a stochastic dynamics is not determined.

$\Rightarrow$ Trajectories of stochastic dynamics can intersect. Nearby trajectory segments point to different directions.

We quantify this criterion of distinction with the nonlinear prediction error.

Nonlinear prediction error

SIMULATION

- Nonlinear prediction error
  - Lorenz
  - Stochastic signals with different correlations
  - Noisy Lorenz
- Influence of time delay τ

Nonlinear prediction error - results

The nonlinear prediction error shows:

- Zero values for periodic dynamics
- Low values for deterministic dynamics
- High values for white noise
- Intermediate values for dynamics that are
  - noisy deterministic
  - noisy periodic
  - correlated stochastic
- Strong dependence on time delay τ

Nonlinear prediction error - Power and limitations

The nonlinear prediction error is sensitive to deterministic structure without being fully specific:

- Strength: detects deterministic structure
- Strength: distinguishes deterministic dynamics and white noise
- Limitation: overlapping results for noisy deterministic and correlated stochastic signals
- Limitation: strong influence of linear correlations

We need a further ingredient:

$\Rightarrow$ The concept of surrogates

The concept of surrogates

- Surrogate signals: very powerful and versatile tool in signal analysis.
- Allow to test a variety of well-defined null hypotheses about the dynamics underlying a signal.
- Essential to judge results of nonlinear signal analysis measures.
- We will introduce them empirically using the example of phase randomized surrogates.
- Null hypothesis: Stationary, linear, stochastic, Gaussian, correlated noise.

Phase randomized surrogates - algorithm

Algorithm for the generation of phase randomized surrogates

- Compute the Fourier transform of the signal
- Randomize all phase angles of the Fourier coefficients
- Compute the inverse Fourier transform of the randomized Fourier coefficients

true positive & true negative null hypothesis rejections

SIMULATION

- Nonlinear prediction error for original signal versus phase randomized surrogates
  - Lorenz dynamics
  - Noisy Lorenz dynamics
  - Stochastic signals
    - Influence of autocorrelation

Outcome

Nonlinear prediction error for original signals and phase randomized surrogates

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coincides for

- Stochastic signals (regardless of autocorrelation)

⇒ null hypothesis not rejected

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does not coincide for

- Lorenz dynamics
- Noisy Lorenz dynamics

⇒ null hypothesis rejected

Interpretation

Combination of the nonlinear prediction error with phase randomized surrogates ⇒ more specific for deterministic structure than the nonlinear prediction error alone.

If null hypothesis is rejected ⇒ some indication that dynamics is non-random.

But careful:

- If the null hypothesis is rejected ⇒ no proof that dynamics is indeed deterministic.
  - Likewise dynamics could be non-stationary, nonlinear stochastic, non-Gaussian, etc.
- If the null hypothesis is not rejected ⇒ no proof that it is indeed correct.
  - Likewise test statistics might not be sensitive enough to detect deterministic structure, nonlinearity, non-stationarity, non Gaussianity, etc.

not a false positive null hypothesis rejection

SIMULATION

- Nonlinear prediction error for original signal versus phase randomized surrogates
  - Non-stationary signals

So what?

Why should the Bern Network for Epilepsy, Sleep and Consciousness care about these results?