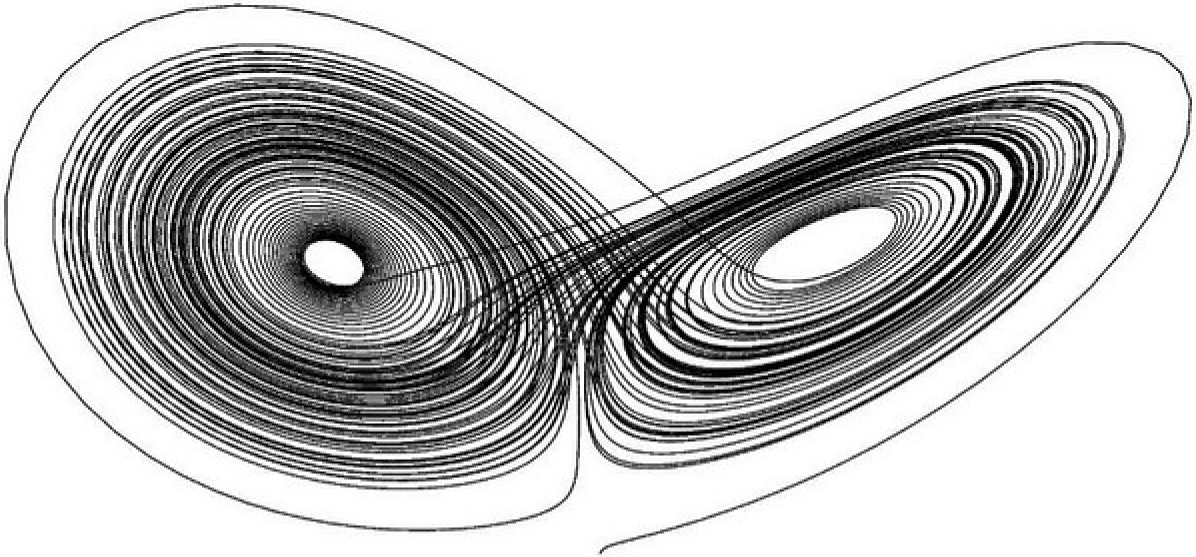
BENESCO Lecture Series on
Sleep, Epilepsy,
Consciousness and Cognition

Bern, Feb. 28th 2020

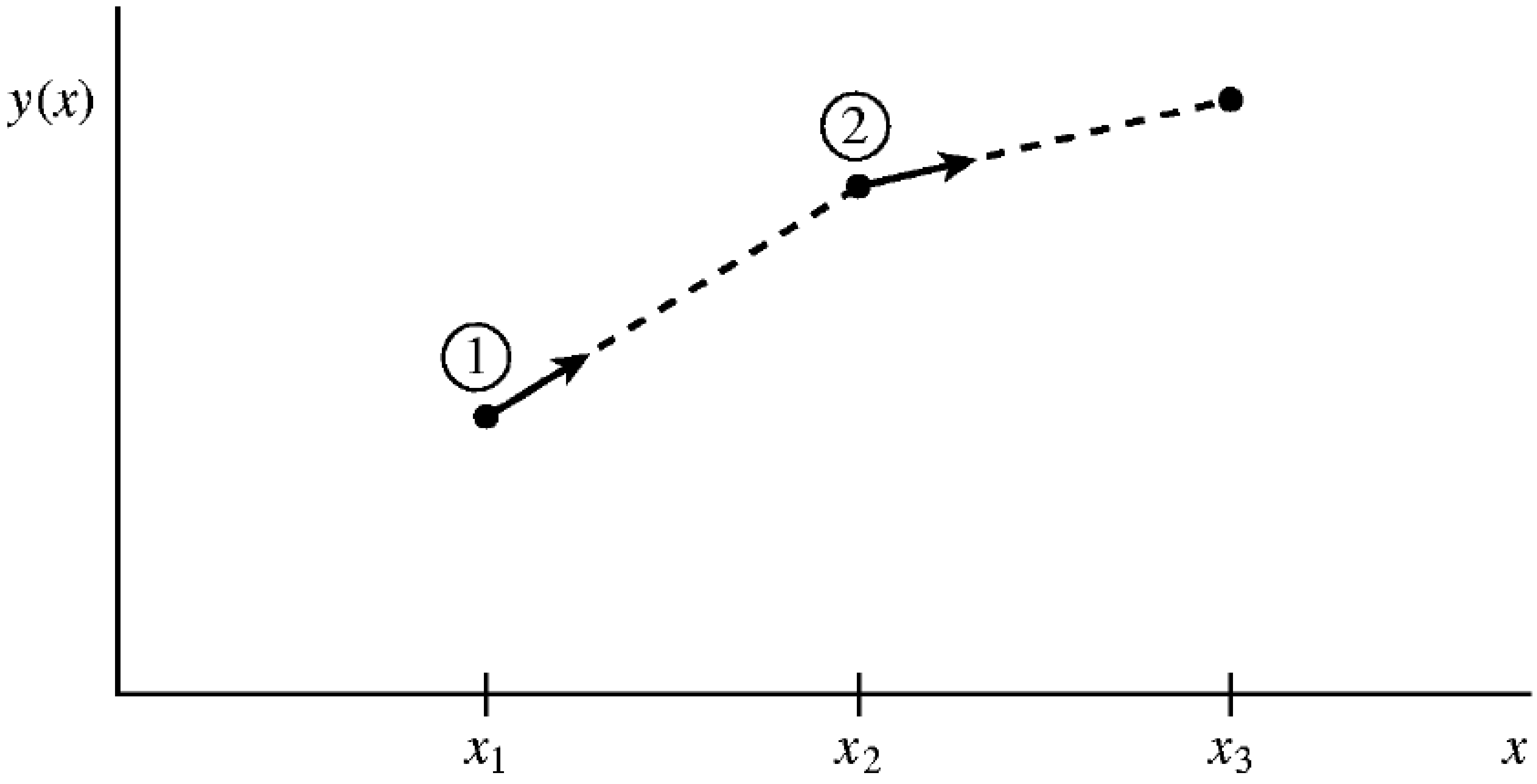
Simple differential equations - an introduction using Matlab



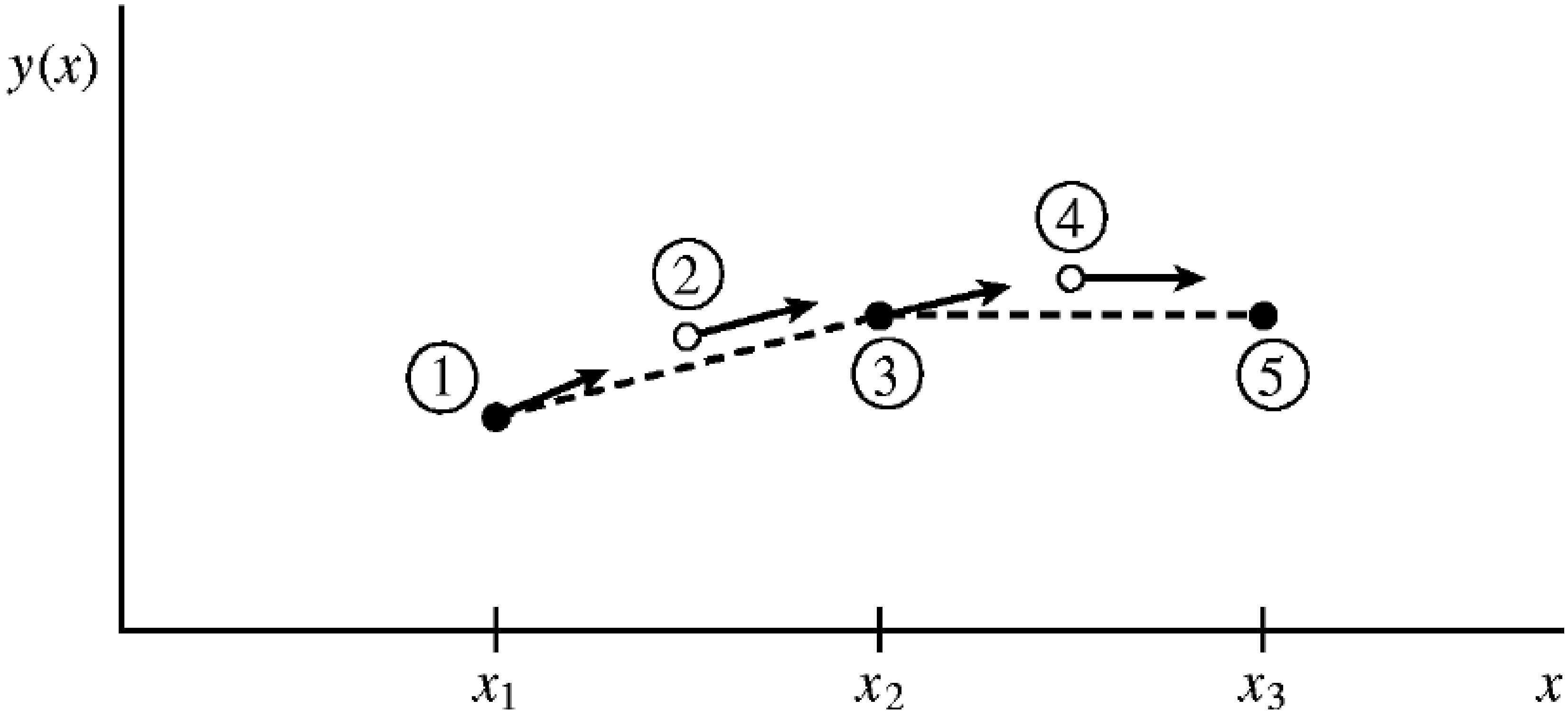
INSELSPITAL
UNIVERSITÄTSSPITAL BERN
HOPITAL UNIVERSITAIRE DE BERNE
BERN UNIVERSITY HOSPITAL

Christian Rummel
SCAN, University Institute of Diagnostic and Interventional Neuroradiology
University of Bern, Inselspital
christian.rummel@insel.ch

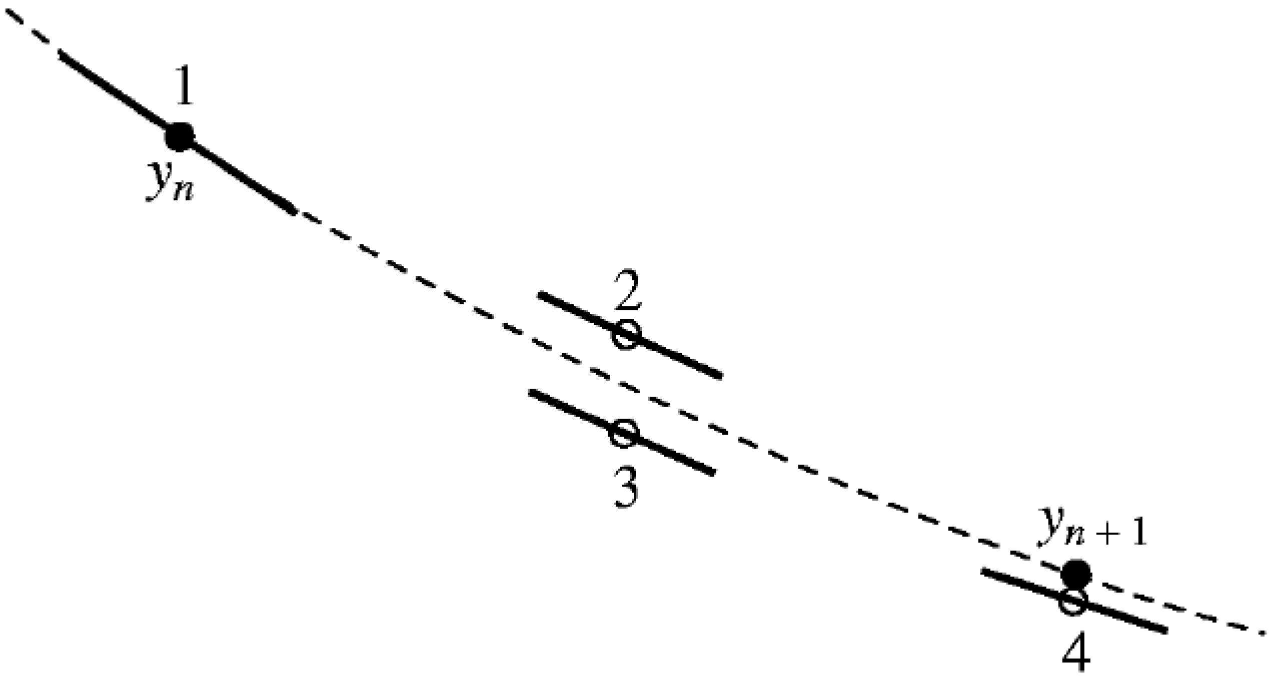
Euler Algorithm (1768–70)



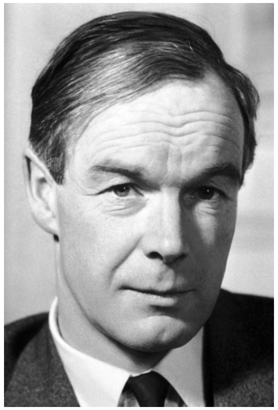
2nd Order Runge-Kutta Algorithm (~1900)

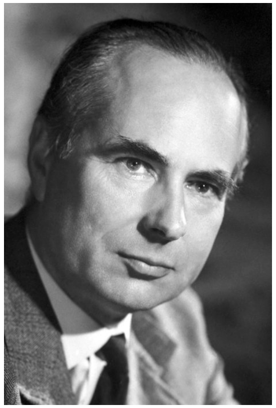


4th Order Runge-Kutta Algorithm (~1900)



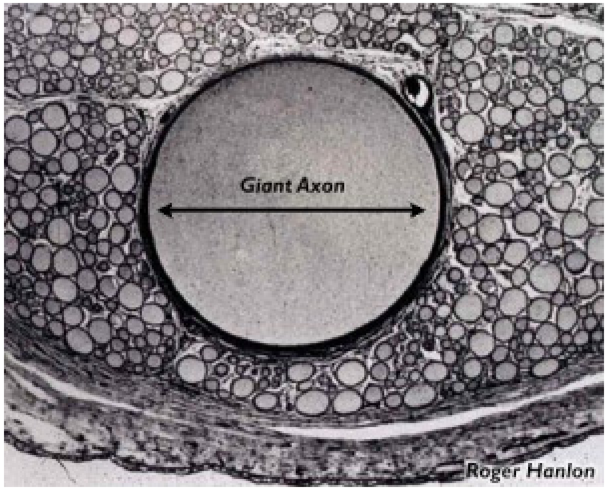
Hodgkin-Huxley Model (1952)

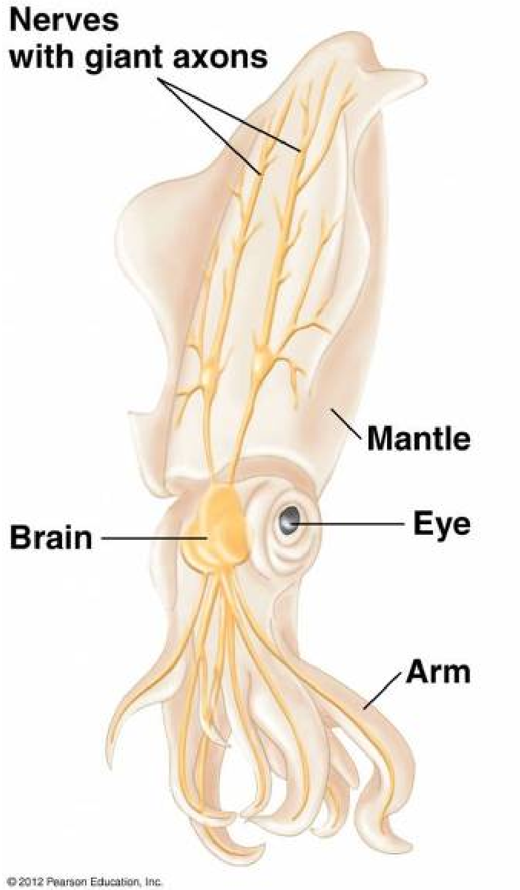




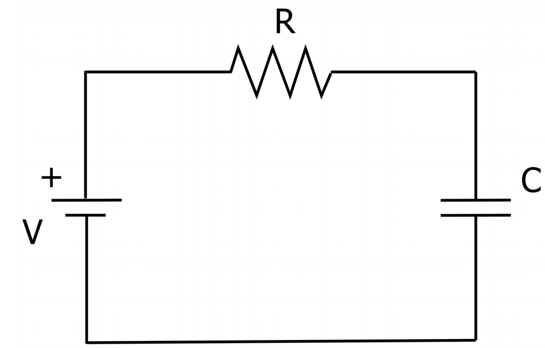
Nobel Prize 1963

Hodgkin-Huxley Model (1952)

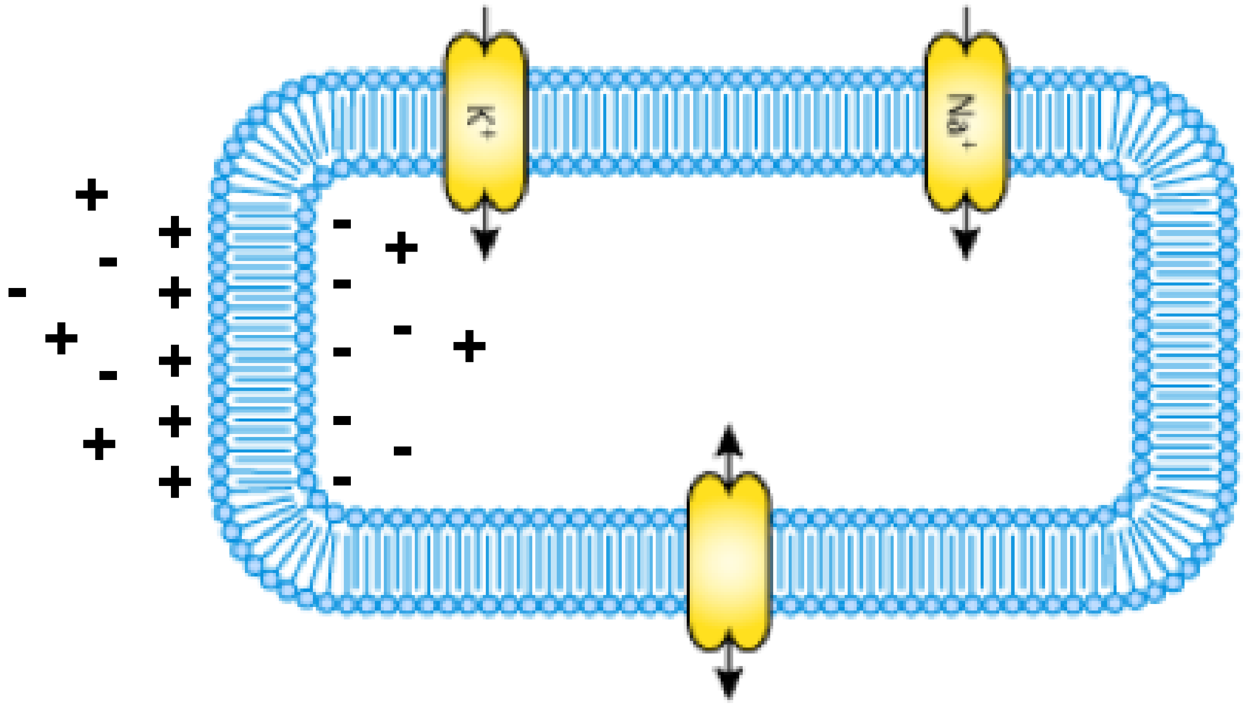




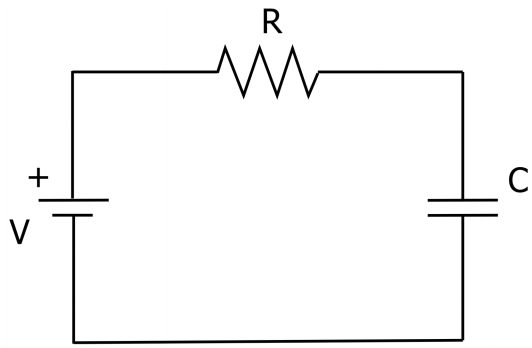
Hodgkin-Huxley Model (1952)



potential across a cell membrane:



Hodgkin-Huxley Model (1952)



voltage variable V:

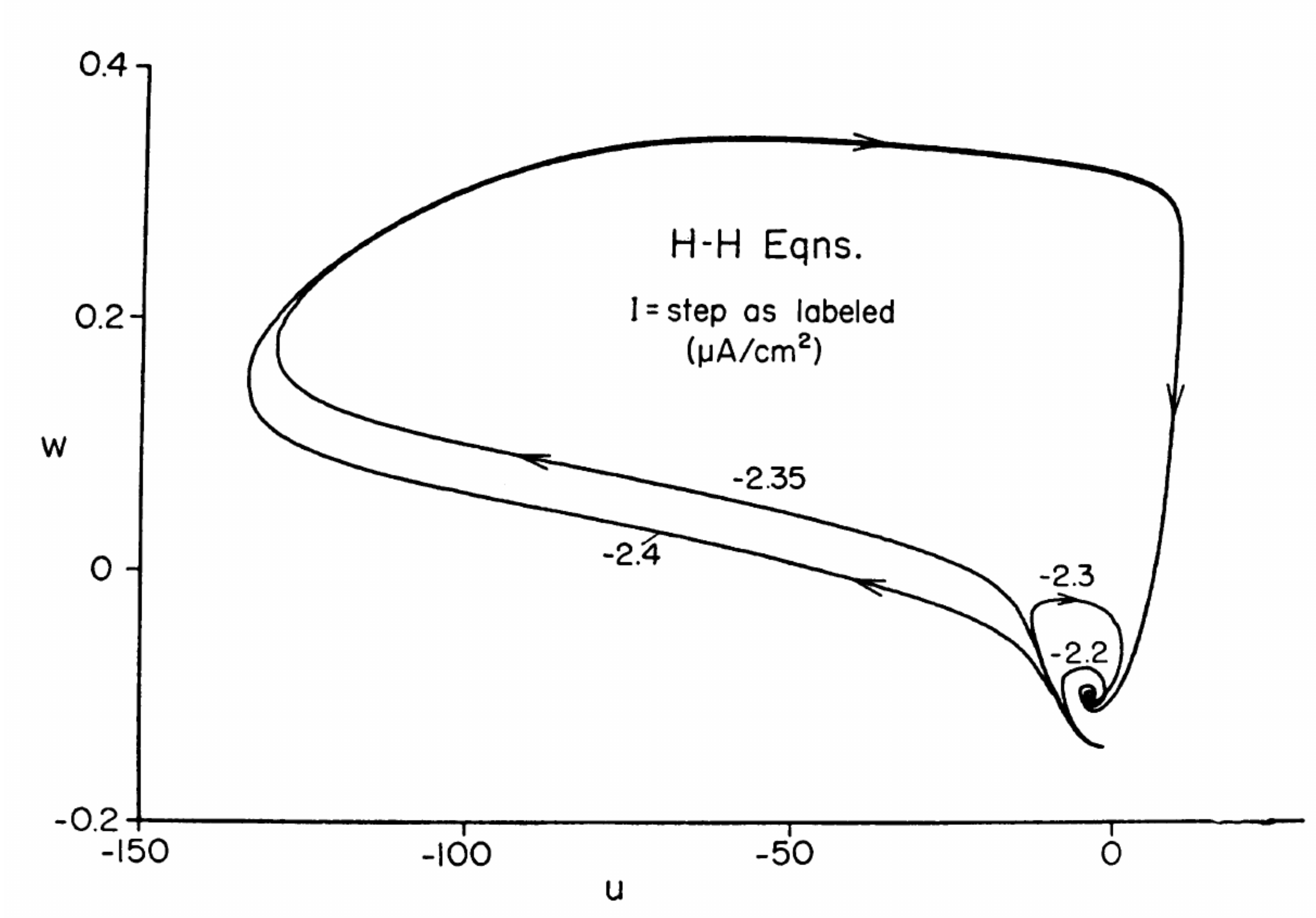
$$C_m \frac{dV}{dt} = -g_K n^4 (V - V_K) - g_{Na} m^3 h (V - V_{Na}) - g_L (V - V_L) + I_{appl}.$$

potassium sodium leakage applied

gating variables m, n, h:

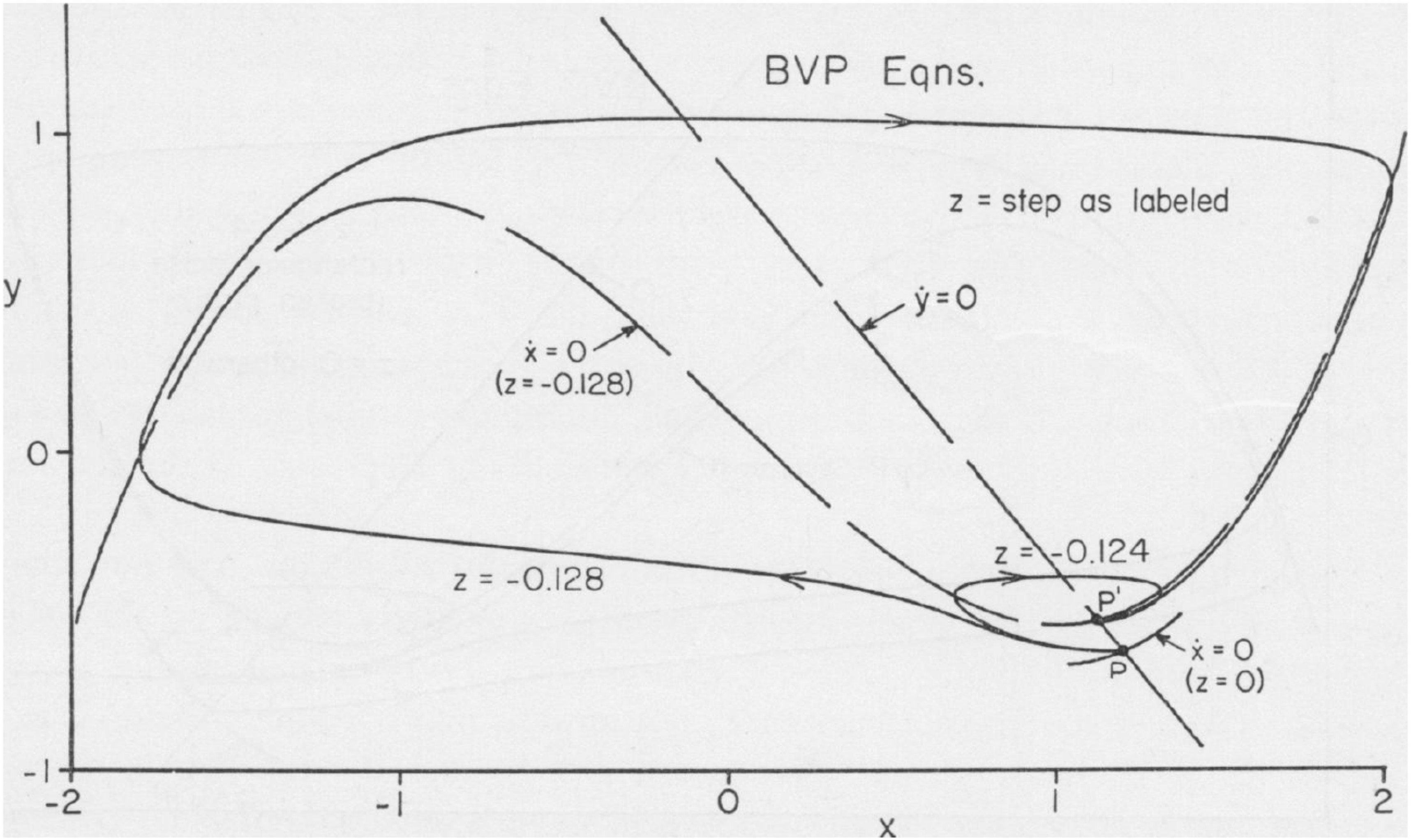
$$\tau_w(V) \frac{dw}{dt} = w_\infty(V) - w, \;\; w = n, m, h,$$

Hodgkin-Huxley Model (1952)



FitzHugh, Biophys J 1 (1961)

FitzHugh-Nagumo Model (1961, 1962)



“Bonhoeffer-van-der-Pol (BVP)” model

FitzHugh, Biophys J 1 (1961)

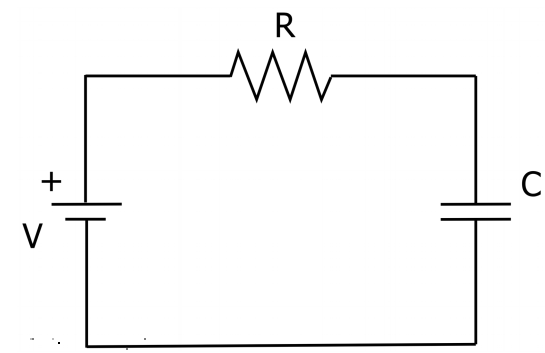
FitzHugh-Nagumo Model (1961, 1962)

- separate slow (h,n) and fast (m) gating variables
- assume linear (n) and cubic (V) form of null clines
- transform to unit-less variables v and w

$$\frac{dv}{dt} = v(v-\alpha)(1-v) - w + I$$
$$\frac{dw}{dt} = \varepsilon(v - \gamma w).$$

FitzHugh, Biophys J 1 (1961)

FitzHugh-Nagumo Model (1961, 1962)



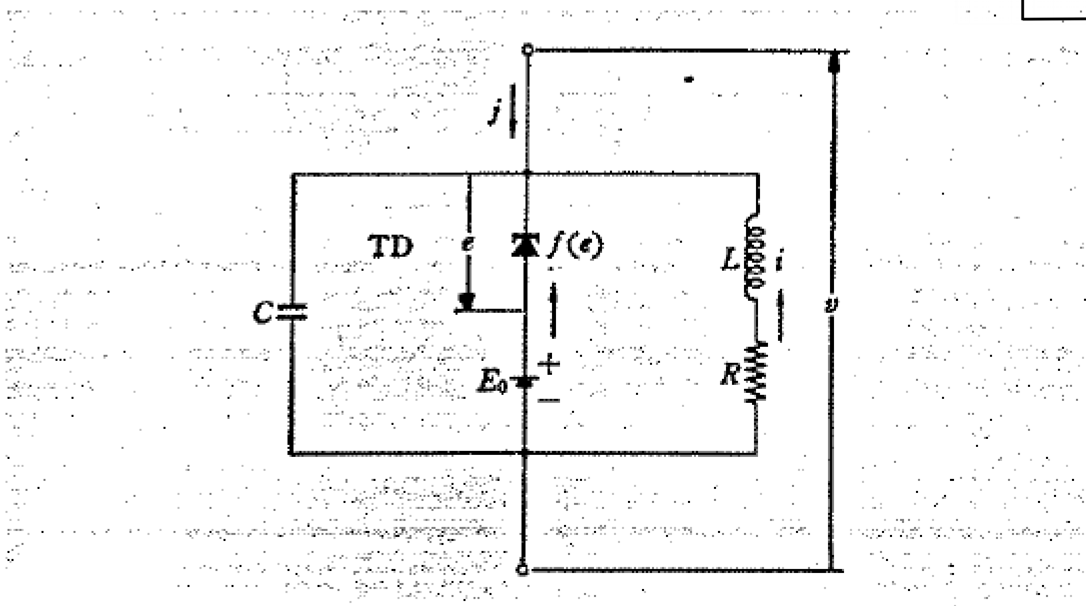


Fig. 2—An electronic simulator of the BVP model.

Nagumo et al., Proc IRE 50 (1962)

Literature

W.H. Press, S.A. Teukolsky, W.T. Vetterling, B.P. Flannery: *Numerical Recipes in C – The Art of Scientific Computing,* 2nd edition, Cambridge University Press, 1992

http://lpsa.swarthmore.edu/NumInt/NumIntIntro.html

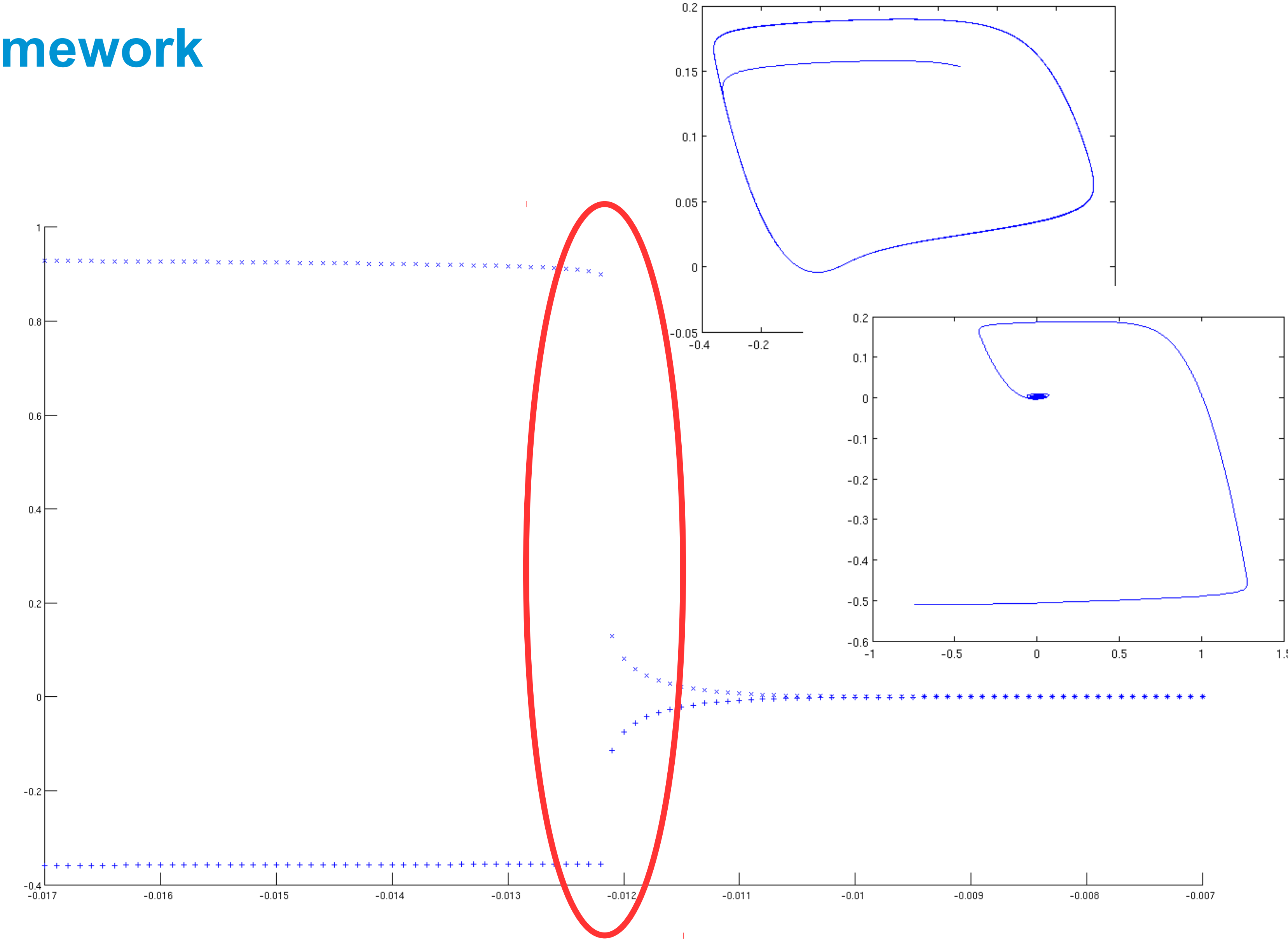
Homework

Appearance of a stable limit cycle in the FitzHugh-Nagumo model (“Hopf bifurcation”):

show numerically that the critical point is $\alpha_0 = -\varepsilon \cdot \gamma$

- write a small Matlab script to ramp up the parameter α of the FitzHugh-Nagumo model ($\varepsilon = 0.008$, $\gamma = 1.5$, $v_{eq} = 0$)
- integrate the differential equation numerically and neglect the transient (be wasteful here!)
- plot the extrema of the v coordinate
- plot example trajectories below and above the bifurcation

Homework


0.2
0.15
0.1
0.05
0
-0.05
-0.4
-0.2
0.2
0.1
0
-0.1
-0.2
-0.3
-0.4
-0.5
-0.6
-1
-0.5
0
0.5
1
1.5
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.017
-0.016
-0.015
-0.014
-0.013
-0.012
-0.011
-0.01
-0.009
-0.008
-0.007