Time-frequency Representation of Brain Oscillations

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BENESCO Lecture Series on
Sleep, Epilepsy, Consciousness and Cognition

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Bern, 29 March 2019

Outline

- Introduction
- Stationarity
- Frequency domain analysis
  - spectral leakage
  - windowing
- Time-frequency signal analysis
  - short-time Fourier transform
  - wavelet analysis
- Spectral analysis of two time series

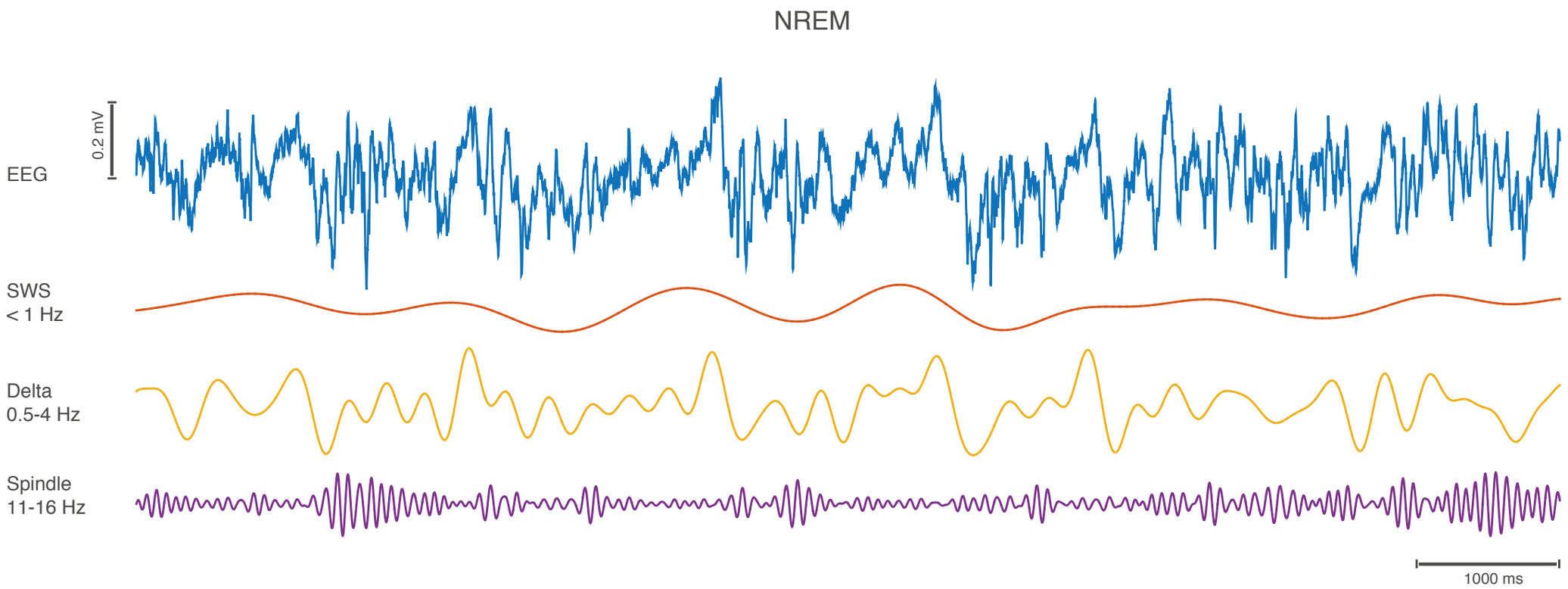
Introduction

- Signals can be treated in
  - Time domain
    - biological signals (EEG, ECG, EMG, MEG, …) are function of time
    - amplitude, variance, autocorrelation, cross-correlation, autoregressive coefficients, zero-crossing, …
  - Frequency domain
    - mostly distinguished information is hidden in frequency contents
    - spectral density estimation, Fourier transform, …
  - Time-frequency domain
    - time-varying signals, such as brain signals
    - short-time Fourier transform, wavelet analysis

Stationarity vs. non-stationarity

- Stationary time series have constant properties over time
  - frequency, mean, variance, autocorrelation, … are constant
  - frequency domain analysis is suitable for stationary signals
- Brain is a dynamic complex system
  - amplitude and frequency content of brain signals change over time
  - neural signals are non-stationary
- How to deal with non-stationary signals?
  - segmentation and averaging
  - time-frequency representations

Frequency domain analysis

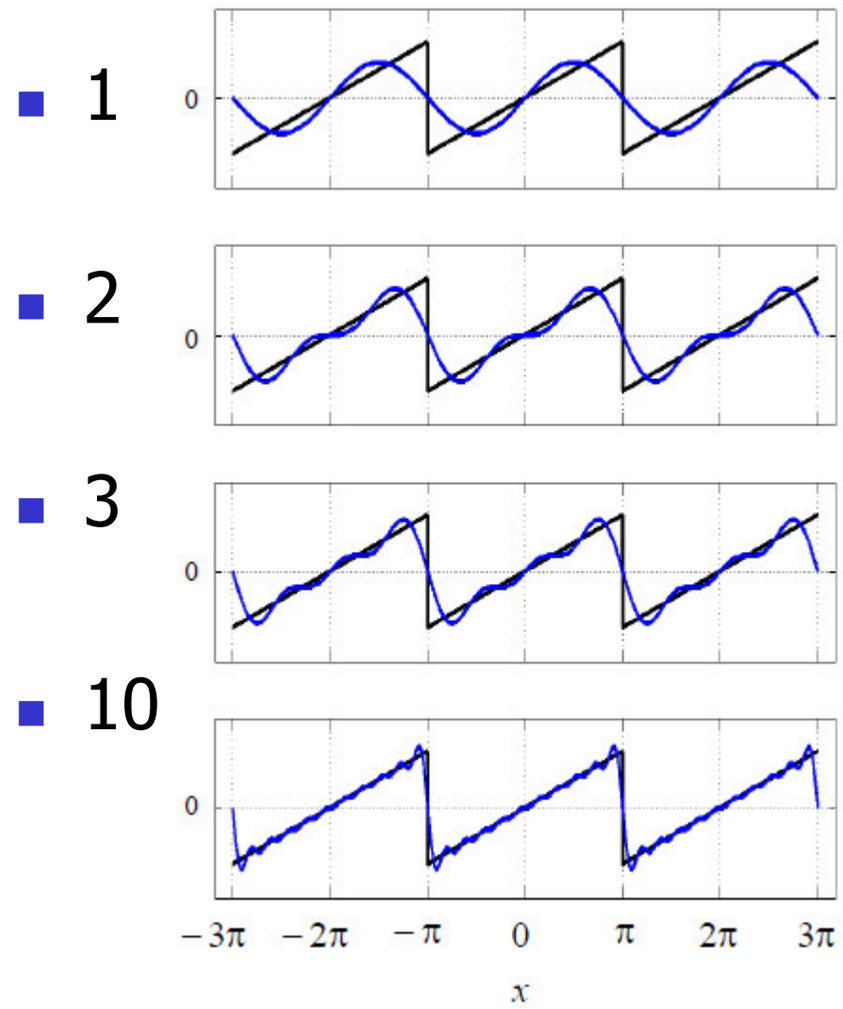


10 s of EEG signal of mouse during non-REM sleep.
Filtered for SWS (Slow Wave Sleep), Delta activity and Spindling

Discrete Fourier Transform (DFT)

- Any waveform can be expressed as a weighted sum of sine/cosine waves!
- DFT provides magnitude and phase at each frequency.

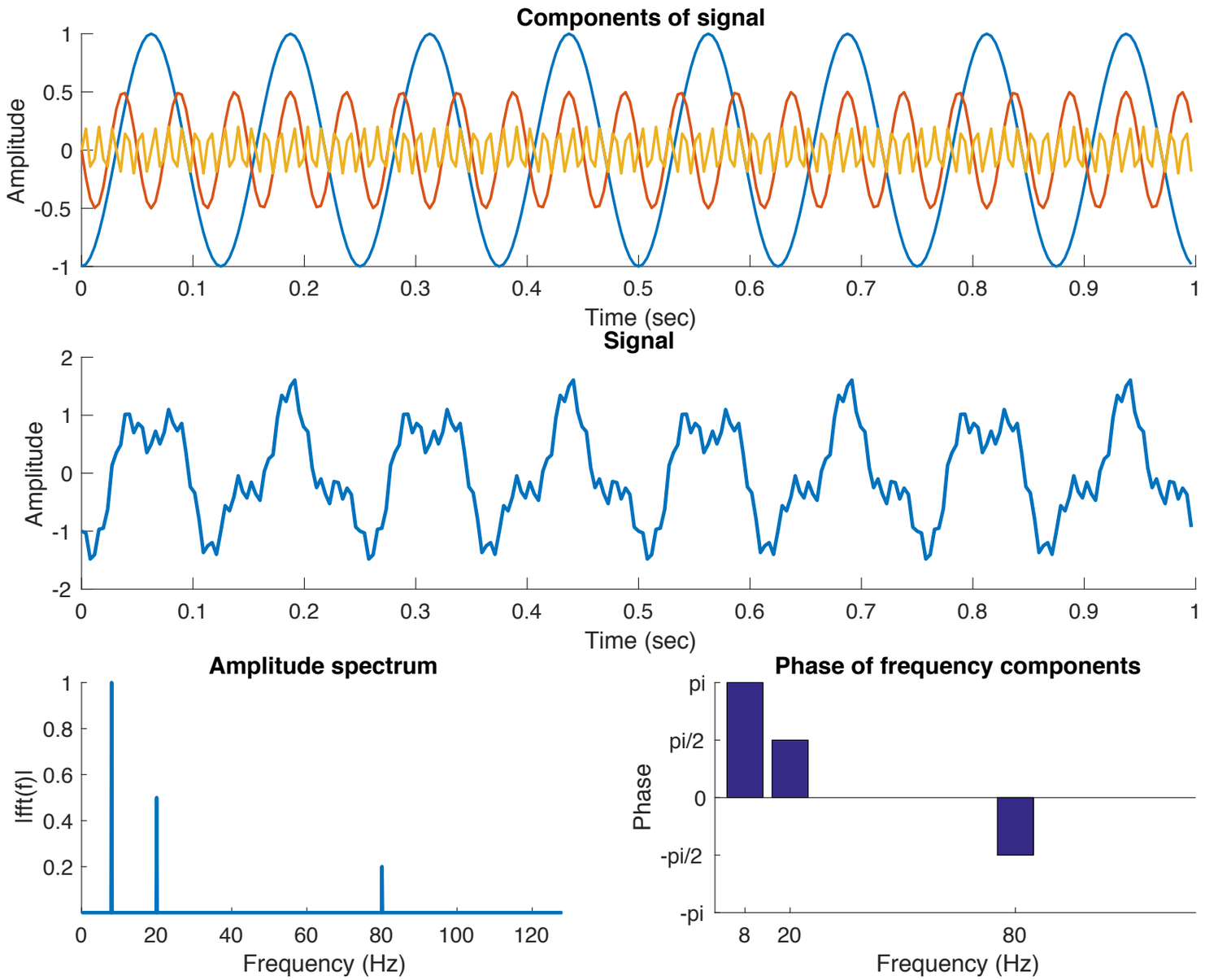
$$X_k \stackrel{\text{def}}{=} \sum_{n=0}^{N-1} x_n \cdot e^{-2\pi i k n / N}, \quad k \in \mathbb{Z}$$



Representing a sawtooth as sum of 10 sine waves.
Source: Wikipedia

Discrete Fourier Transform (DFT)

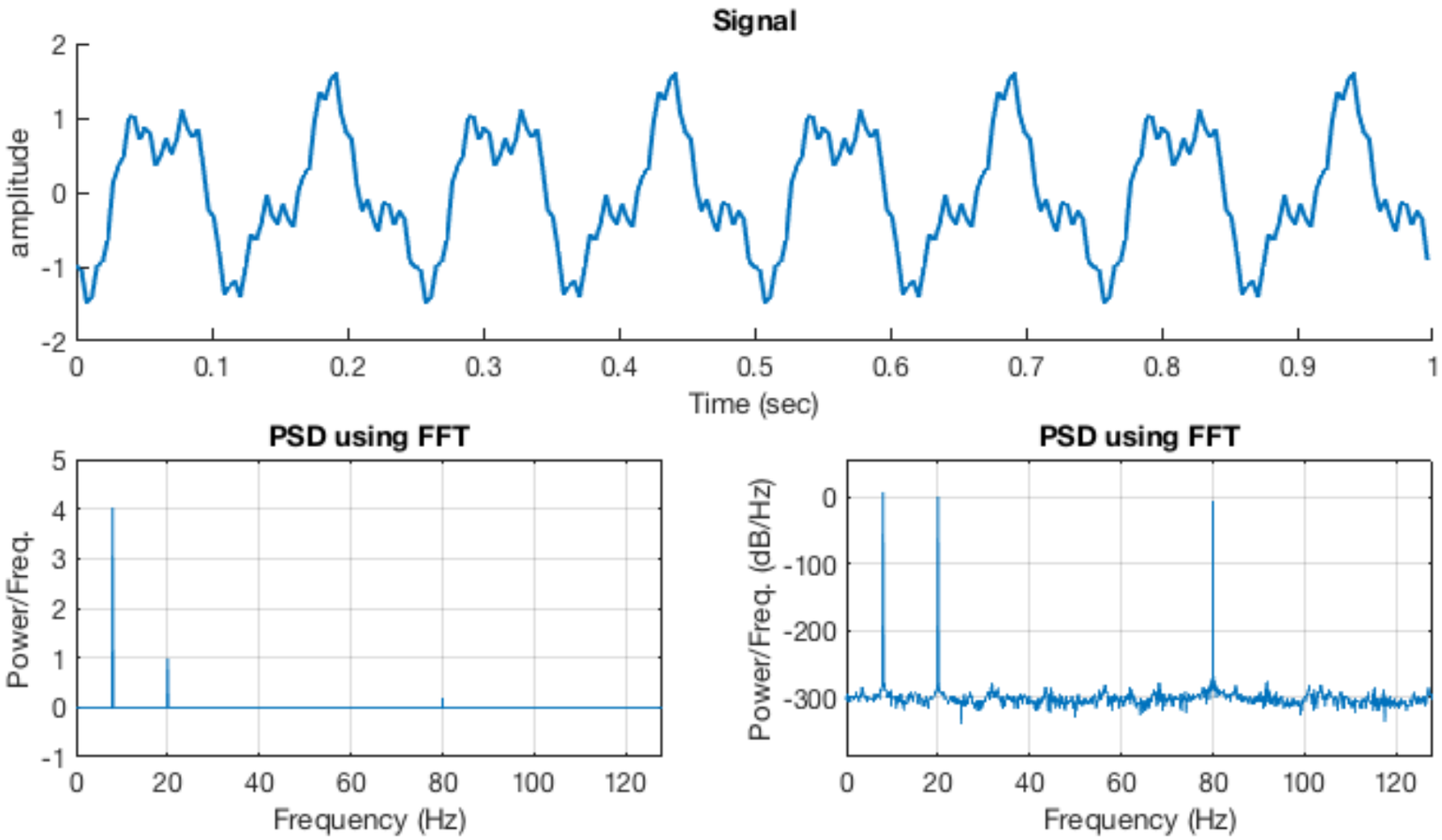
```
% MATLAB

% 8 Hz with pi phase
comp1 = 1.0*cos(2*pi*8*time+pi);
% 20 Hz with +pi/2 phase
comp2 = 0.5*cos(2*pi*20*time+pi/2);
% 80 Hz with -pi/2 phase
comp3 = 0.2*cos(2*pi*80*time-pi/2);
% Merge components
signal = comp1 + comp2 + comp3;

% and now Fast Fourier Transform (FFT)
fftSig = fft(signal);
fftSig = fftSig(1:lenSig/2+1);  % single-sided
freq = 0:sampFreq/lenSig:sampFreq/2;
absFFT = abs(fftSig/lenSig);  % absolute values

absFFT(2:end-1) = 2*absFFT(2:end-1);  %
multiply single-sided spectrum by 2

phaseFFT = angle(fftSig); % phases of freqs.
```



Power spectral density (PSD)

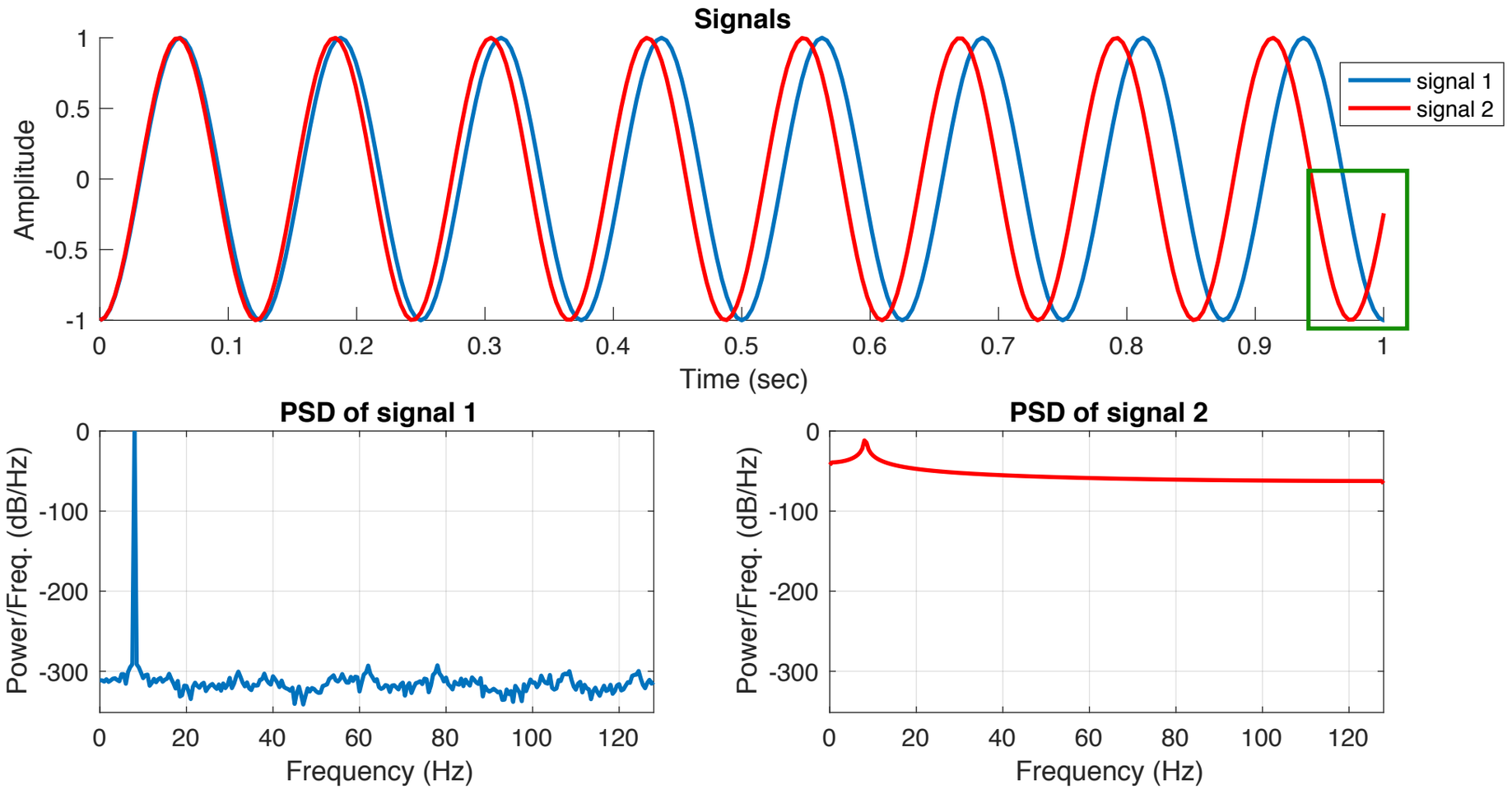
- FFT provides **amplitude** and **phase** of each contributing frequency
- PSD describes how **power** of a signal is distributed over frequencies

```
% MATLAB

% 8 Hz with pi phase
comp1 = 1.0*cos(2*pi*8*time+pi);
% 20 Hz with +pi/2 phase
comp2 = 0.5*cos(2*pi*20*time+pi/2);
% 80 Hz with -pi/2 phase
comp3 = 0.2*cos(2*pi*80*time-pi/2);
% Merge components
signal = comp1 + comp2 + comp3;

% FFT
fftSig = fft(signal);
fftSig = fftSig(1:lenSig/2+1);  % single-sided
freq = 0:sampFreq/lenSig:sampFreq/2;

absFFT = abs(fftSig/lenSig);  % absolute
absFFT(2:end-1) = 2*absFFT(2:end-1);  %
multiply single-sided spectrum by 2

% PSD from FFT
psdFFT = 1/(sampFreq*lenSig) * abs(fftSig).^2;
psdFFT(2:end-1) = 2*psdFFT(2:end-1);
```



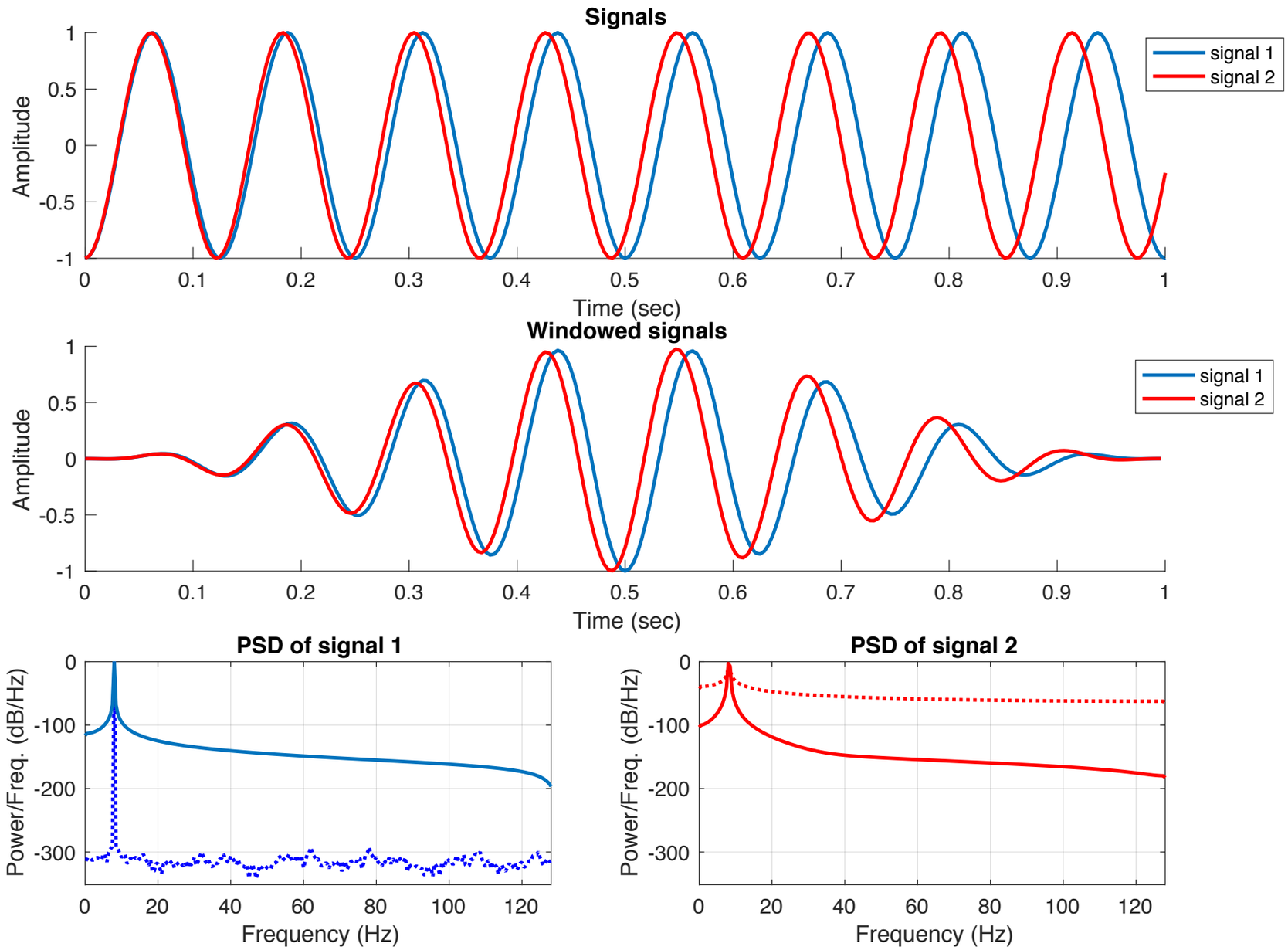
Spectral leakage

- If the number of cycles of a frequency is not an integer (in a segment of data), endpoints are discontinuous.
- These artificial discontinuities show up in the FFT as wide range of frequency components not present in the original signal.



Spectral leakage and windowing

- This is unavoidable for DFT, but can be improved using windowing.

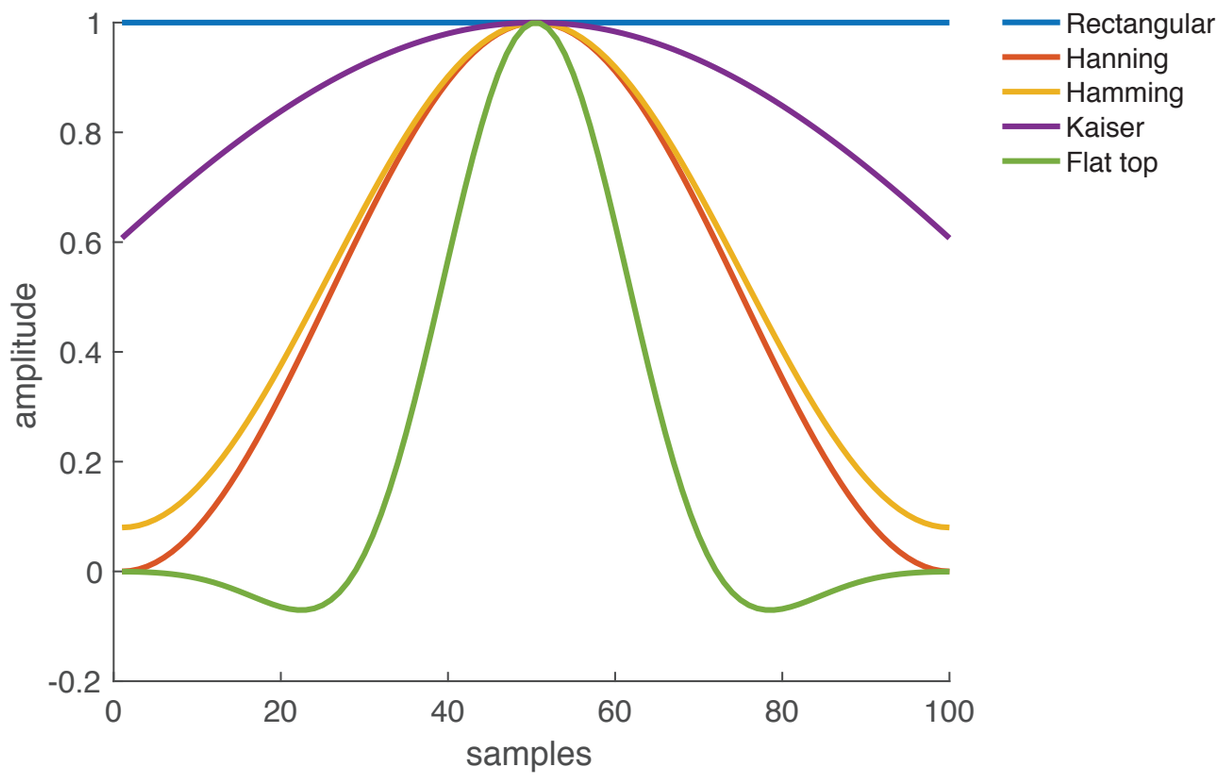


Windowing functions

- There are several windowing functions for different situations/applications
  - Hanning, Hamming, Kaiser, … (see http://ch.mathworks.com/help/signal/windows.html)
  - in general, the Hanning window is satisfactory in 95 percent of cases.

Hanning window: $w_j = \frac{1}{2}\left[1 - \cos\left(\frac{2\pi j}{N-1}\right)\right] \quad j \in [0 \ldots N-1]$

```
% MATLAB

winLen = 100; % length of window in sample

rectWin = window(@rectwin,winLen); % Rectangular

hannWin = window(@hann,winLen); % Hanning

hammWin = window(@hamming,winLen); % Hamming

kaiserWin = window(@kaiser,winLen,1.5); % Kaiser

flattopWin = window(@flattopwin,winLen); % Flat top
```



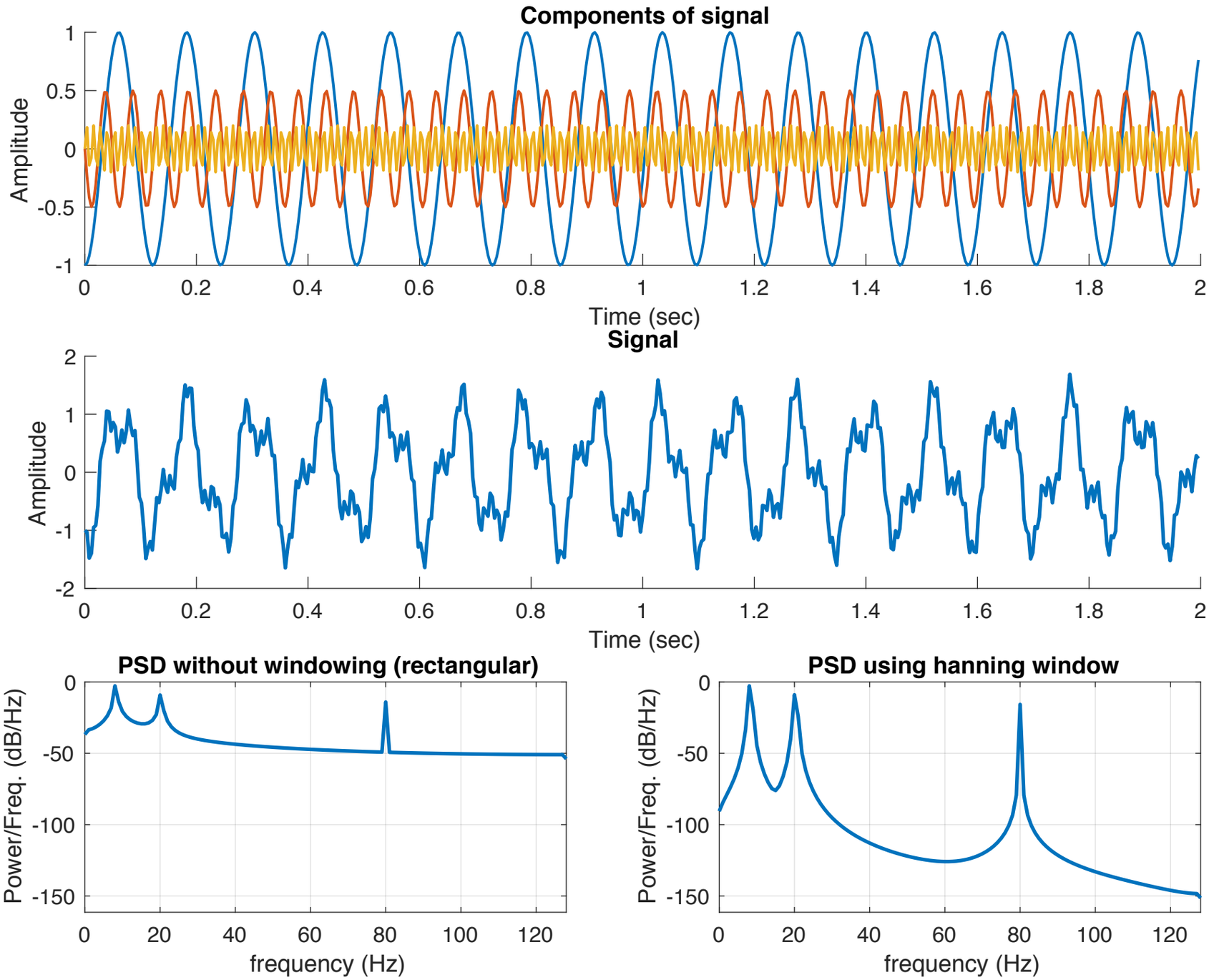
Some of famous windowing functions of length 100 samples

Choice of window

Type of Signal	Window function
Transients whose duration is shorter than the length of the window	Rectangular
Transients whose duration is longer than the length of the window	Exponential, Hanning
General-purpose applications	Hanning
Spectral analysis (frequency-response measurements)	Hanning (for random excitation), Rectangular (for pseudorandom excitation)
Separation of two tones with frequencies very close to each other but with widely differing amplitudes	Kaiser-Bessel
Separation of two tones with frequencies very close to each other but with almost equal amplitudes	Rectangular
Accurate single-tone amplitude measurements	Flat top
Sine wave or combination of sine waves	Hanning
Sine wave and amplitude accuracy is important	Flat top
Narrowband random signal (vibration data)	Hanning
Broadband random (white noise)	Rectangular
Closely spaced sine waves	Rectangular, Hamming
Excitation signals (hammer blow)	Force
Response signals	Exponential
Unknown content	Hanning

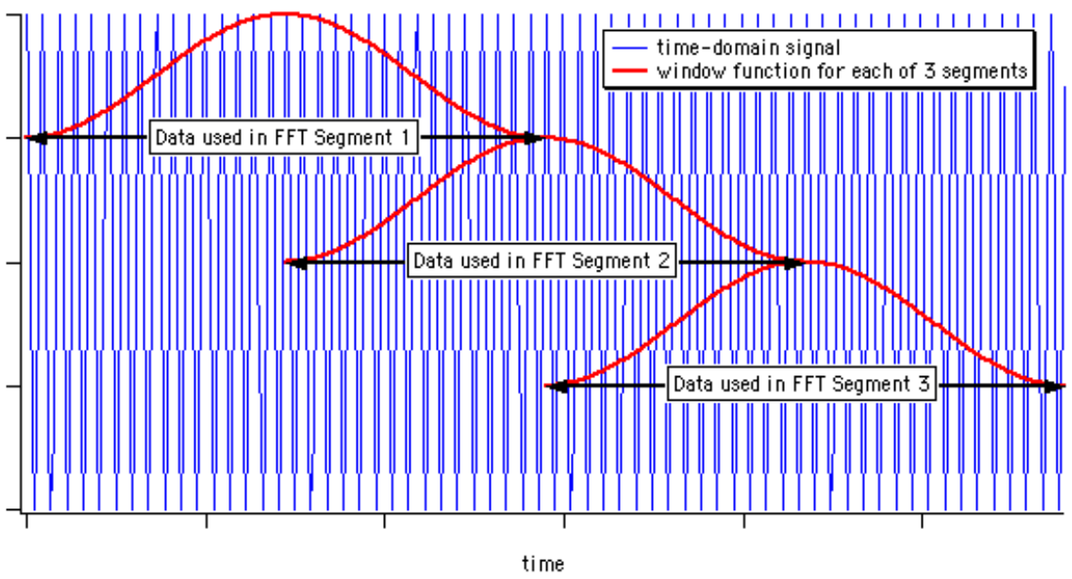
http://zone.ni.com/reference/en-XX/help/372614J-01/lvanlsconcepts/choosing_smoothing_window/

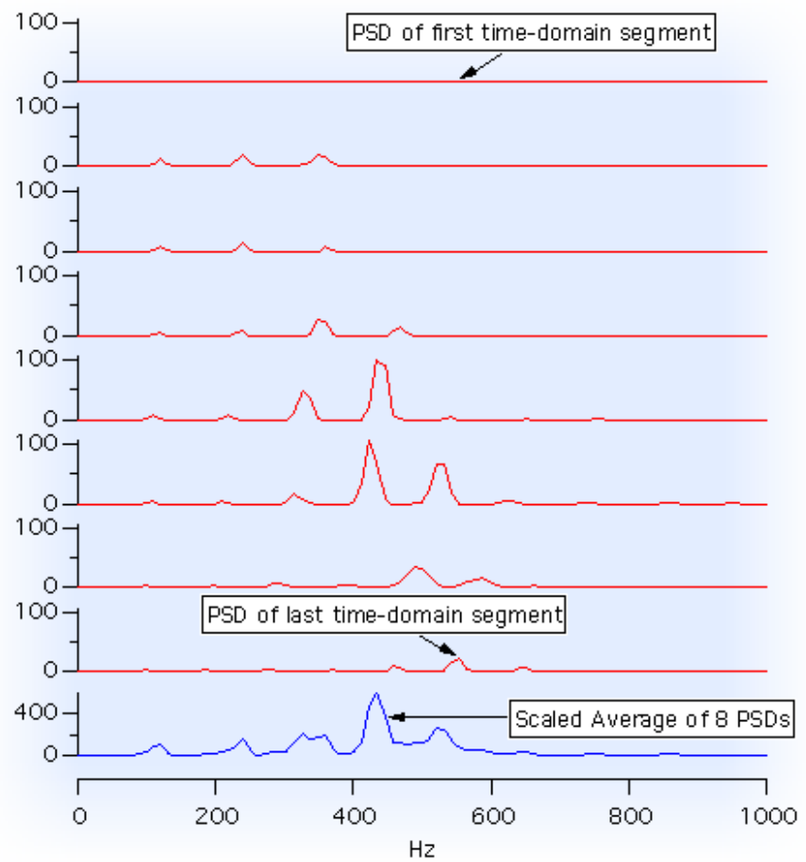
Spectral leakage



PSD of non-stationary and long signals

- **Welch's method** is an improved PSD estimator, reduces noise by averaging
  - split up signal into overlapping segments
  - a window function (such as hamming) is applied on segments
  - squared magnitude of DFT is calculated for each segment
  - individual PSDs are averaged



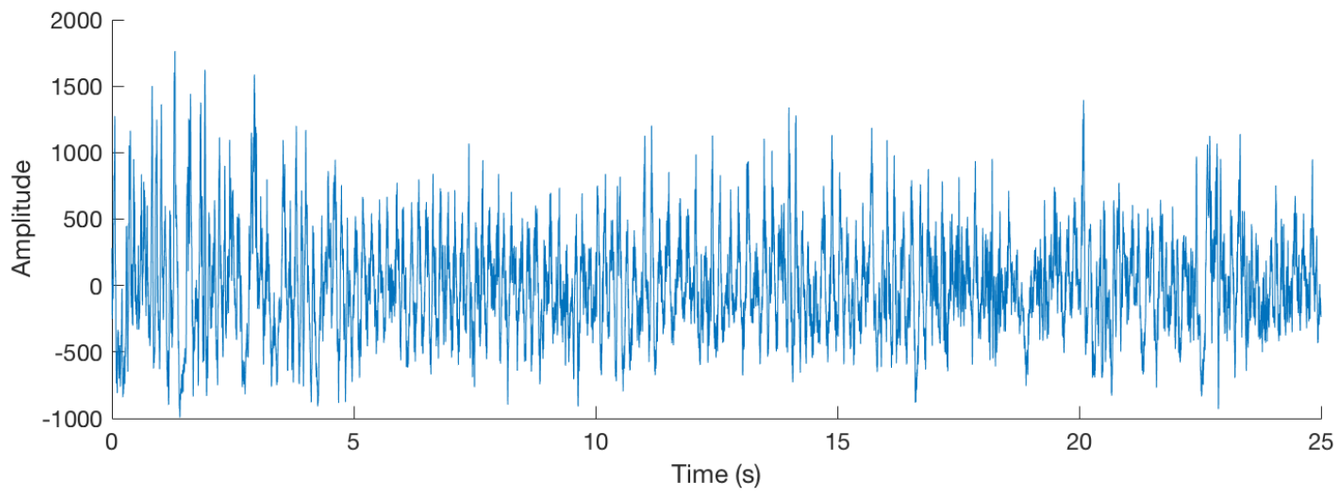


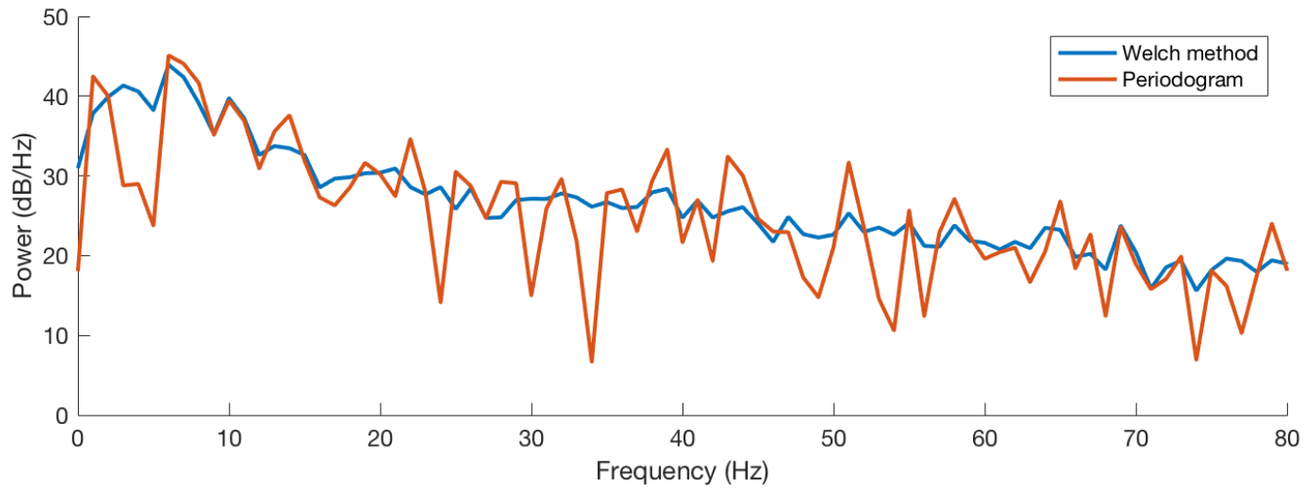
https://www.wavemetrics.com/products/igorpro/dataanalysis/signalprocessing/powerspectra.htm

PSD using Welch's method

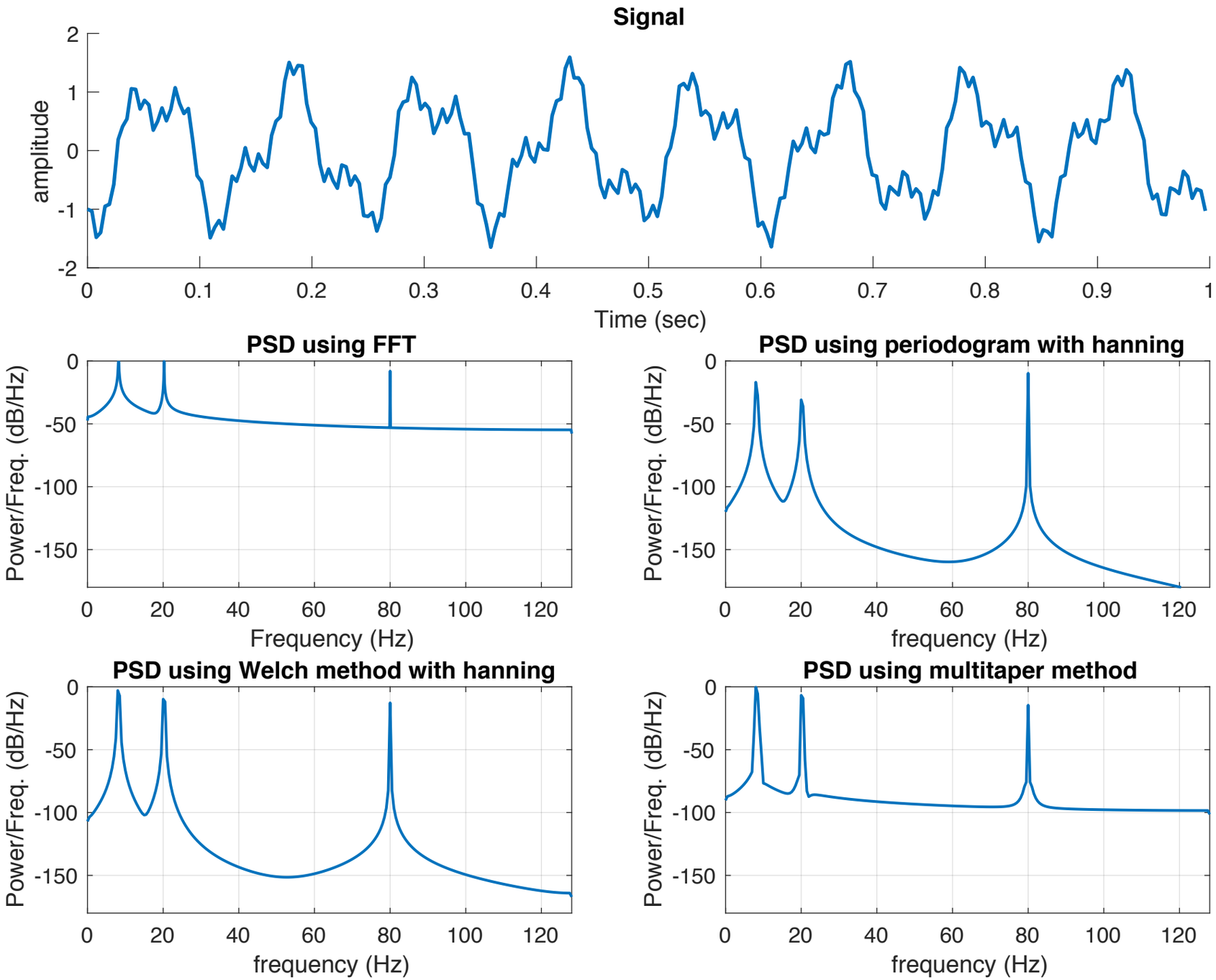
- "pwelch.m" function [pxx,f] = pwelch(x,window,noverlap,f,fs)
- Welch's method vs. periodogram (no windowing)

```
% MATLAB code
% rawSig and fs are provided in lecture notes
% rawSig is hippocampal LFP of mouse (fs = 1000 Hz)
t = 0:1/fs:(length(rawSig)-1)/fs;
subplot(211); plot(t,rawSig); box off
xlabel('Time (s)')
ylabel('Amplitude')

window = 4*fs; % 4-sec windows (resolution 0.25Hz)
overlap = round(0.5*window); % 50% overlap
[pxx,f] = pwelch(rawSig,window,overlap,fs,fs);
subplot(212); hold on
plot(f,10*log10(pxx),'linewidth',2); box off
xlim([0 80])
xlabel('Frequency (Hz)')
ylabel('Power (dB/Hz)')

[pxx1,f1] = periodogram(rawSig,...
    rectwin(length(rawSig)),fs,fs);
plot(f1,10*log10(pxx1),'linewidth',2); hold off
xlim([0 80])
xlabel('Frequency (Hz)')
ylabel('Power (dB/Hz)')
legend({'Welch method','Periodogram'})
```





PSD - comparing some approaches



What to do with FFT and PSD?

- Finding periodicities within neural signals in different states
  - e.g. slow wave and delta of NREM sleep
- Quantifying power of frequencies in different conditions
  - e.g. delta, theta, alpha, beta, gamma, …
  - health vs disease
- Spectral edge frequency and spectral edge power
  - spectral edge frequency: below which x% of total power of signal is located
  - spectral edge power: total power below the spectral edge frequency
- Finding interfering signals (e.g. power line noise)

Conclusion - frequency domain analysis

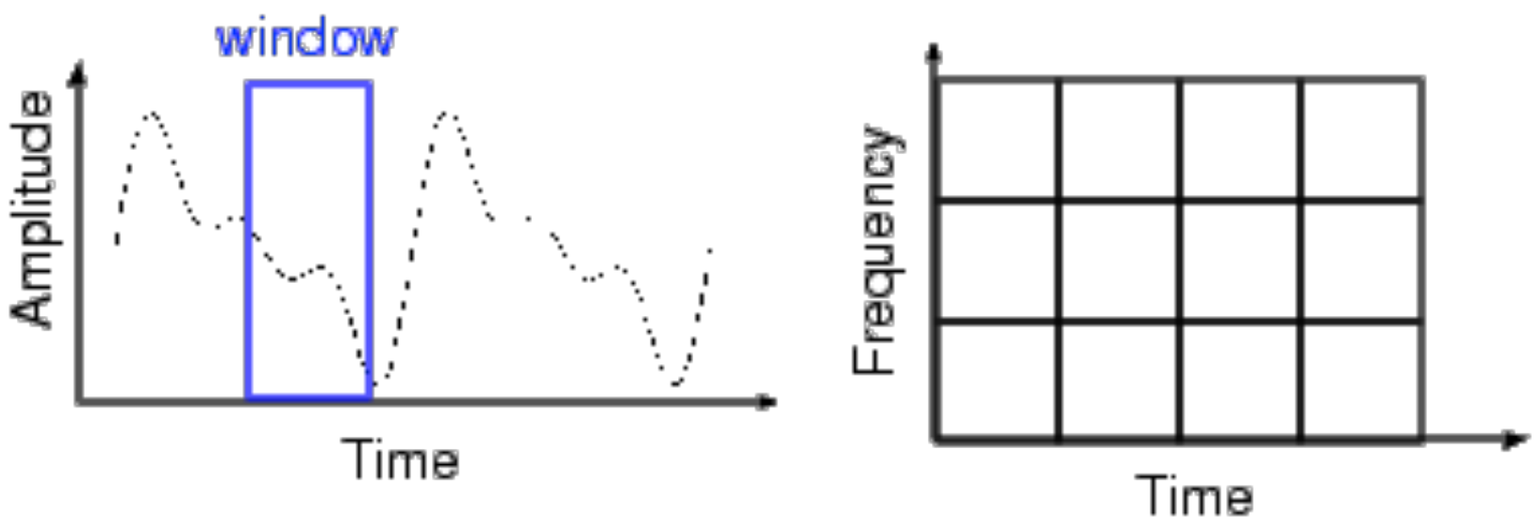
- Useful to have a global view of frequency components of a signal
- Suitable for stationary signals
  - the frequency content doesn’t change with time (time-invariant)
- Loosing time information
  - when do transient changes occur?
  - which frequencies exist at a particular time.

Time-frequency representation (TFR)

- TFR provides information in time-frequency plane
- Some applications in neural signal processing
  - analysis of event-related potential (ERP)
  - neurostimulation response analysis (Electrical / Optogenetics, TMS, …)
  - studying of epileptic seizures
  - recalling a specific memory
- Some approaches
  - **Short-time Fourier transform (spectrogram)**
  - **Wavelet transform (scalogram), wavelet coherence**
  - Wigner distribution
  - Choi-Williams distribution
  - Matching pursuit

Short-Time Fourier Transform (STFT)

- Signals can be assumed quasi-stationary in short times
  - Short time definition depends on frequency content of signal
  - Window length should cover few cycles of main frequency component

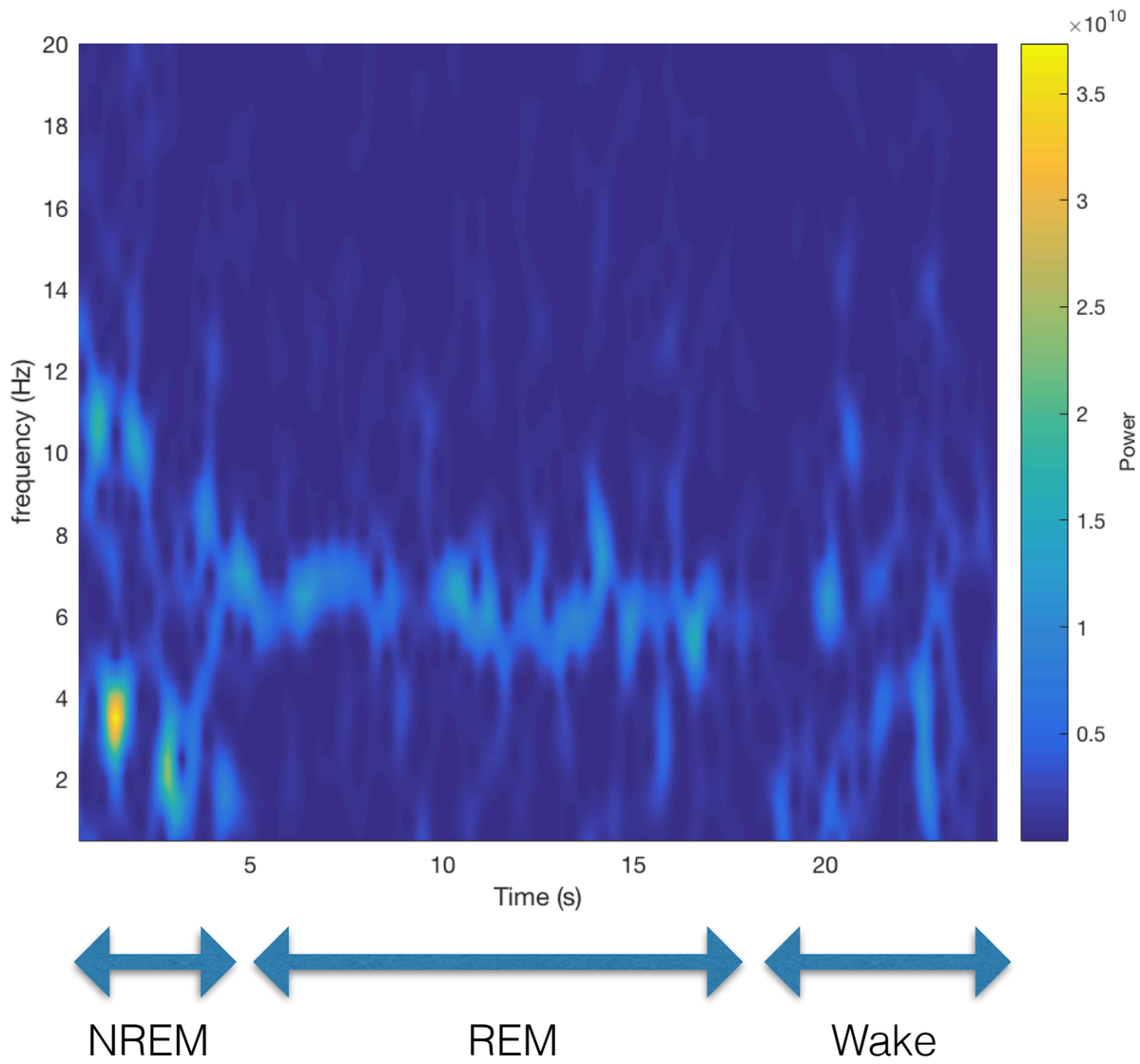


STFT - MATLAB

- “spectrogram.m” function

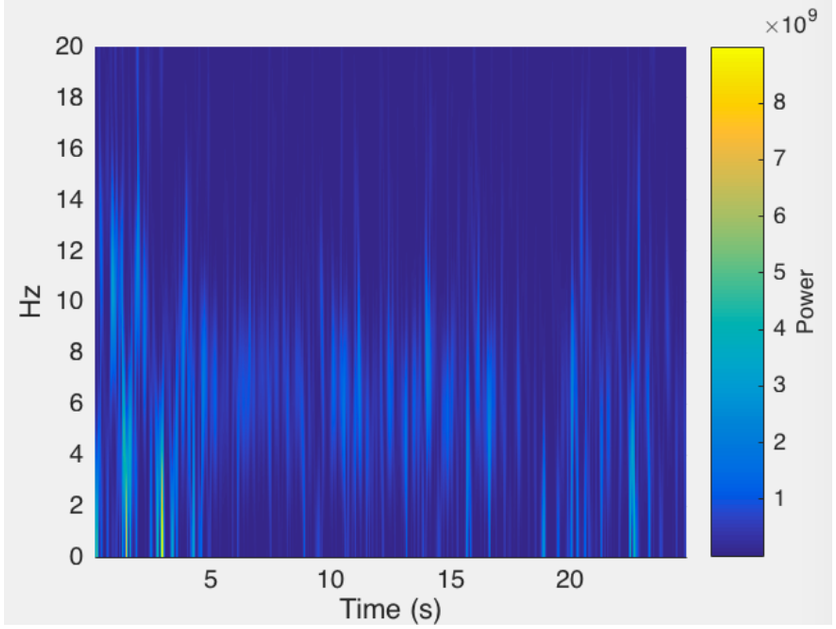
[s,f,t] = spectrogram(x,window,noverlap,f,fs)

```
% MATLAB code
% rawSig and fs are provided in lecture notes
% rawSig is hippocampal LFP from mouse (fs = 1000 Hz)

window = 1*fs; % window size in sample
overlap = round(0.96*window); % overlap is an integer value

[SC,freq,time] = spectrogram(rawSig,window,overlap,2*fs,fs);
SC = abs(SC(freq>=minfreq&freq<=maxfreq,:));
freq = freq(freq>=minfreq&freq<=maxfreq);
args = {time,freq,SC.^2};

figure('Color',[1 1 1]);
surf(args{:},'edgecolor','none');
view(0,90);
axis tight;
shading interp;
colormap(parula(128));
h = colorbar;
h.Label.String = 'Power';
xlabel('Time (s)');
ylabel('frequency (Hz)');
```

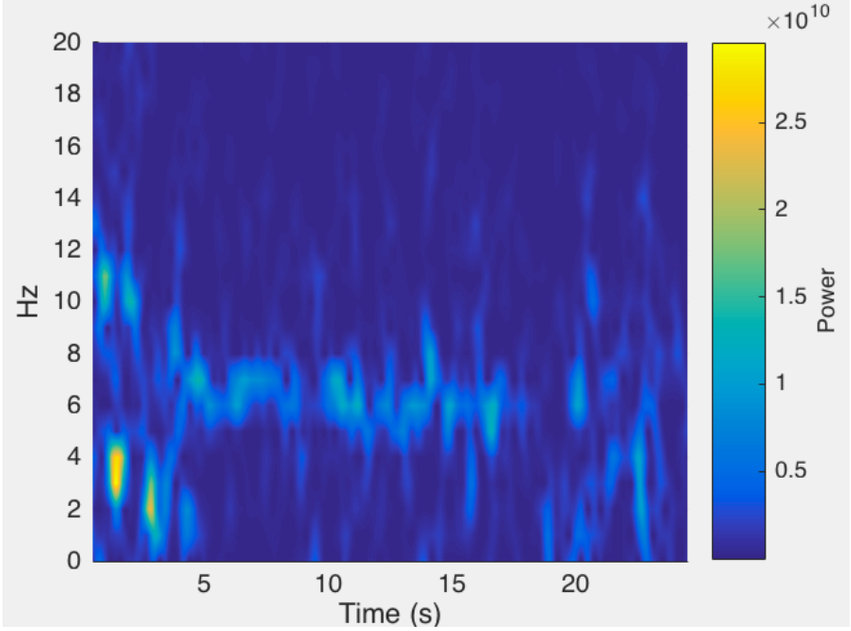


STFT - effect of window length

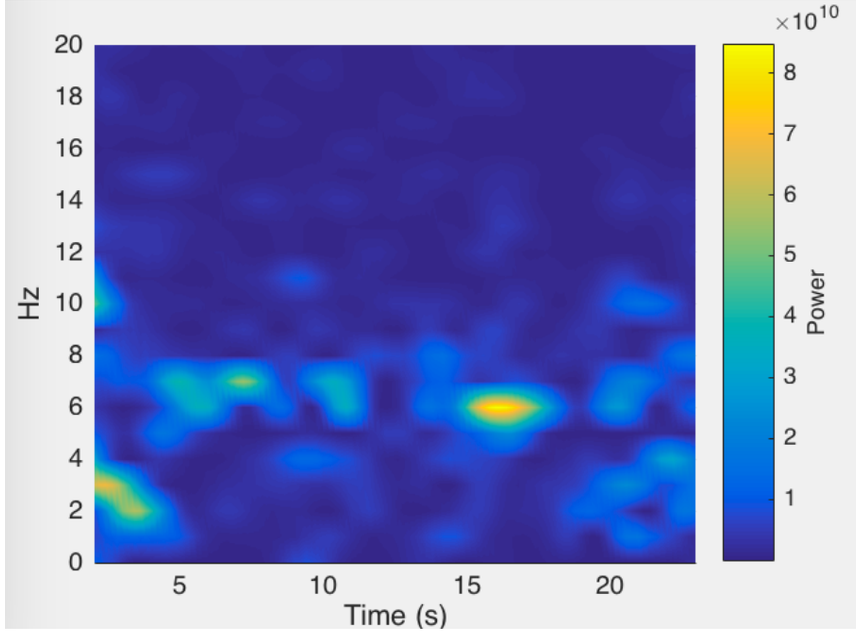
- Tradeoff between time and frequency resolutions
  - a narrow window provides better time resolution, degrade freq. resolution
  - a wide window provides better freq. resolution, degrade time resolution



Window length = 0.25 sec



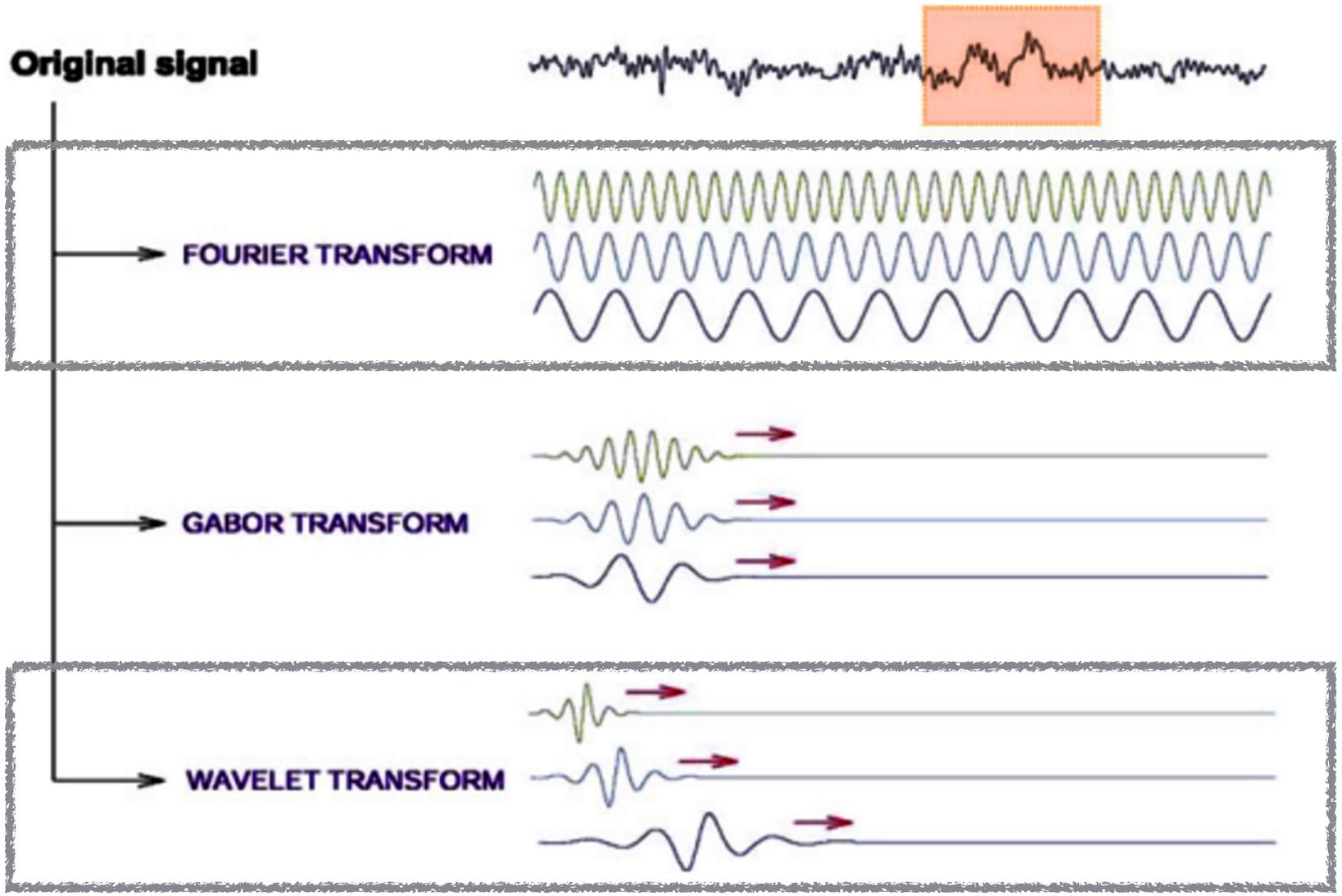
Window length = 1 sec



Window length = 4 sec

Wavelet transform

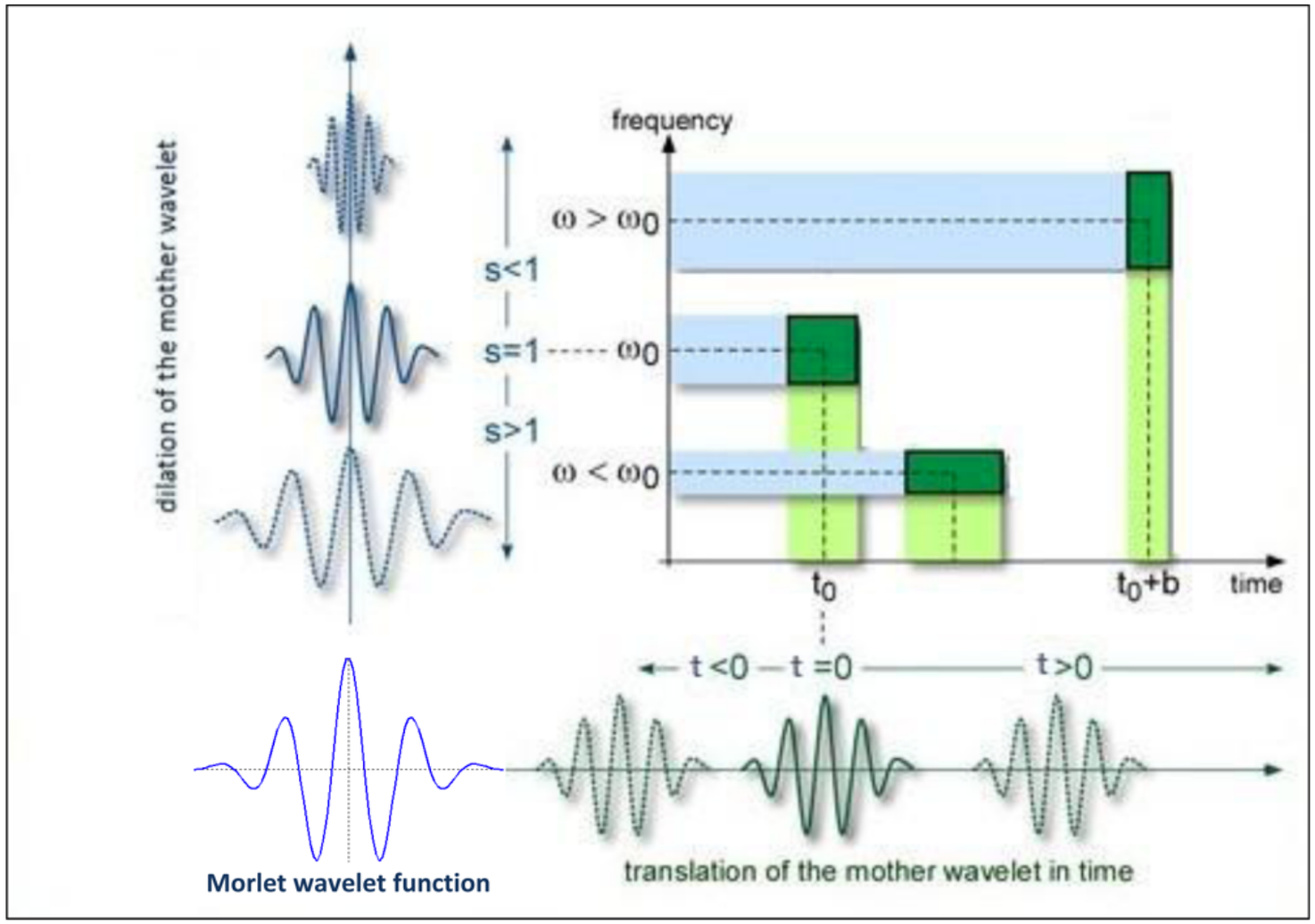
- Convolution between signal and scaled versions of a wavelet function.



Freeman W., Quiroga R. Q., Imaging Brain Function With EEG, Springer, 2013

Wavelet transform

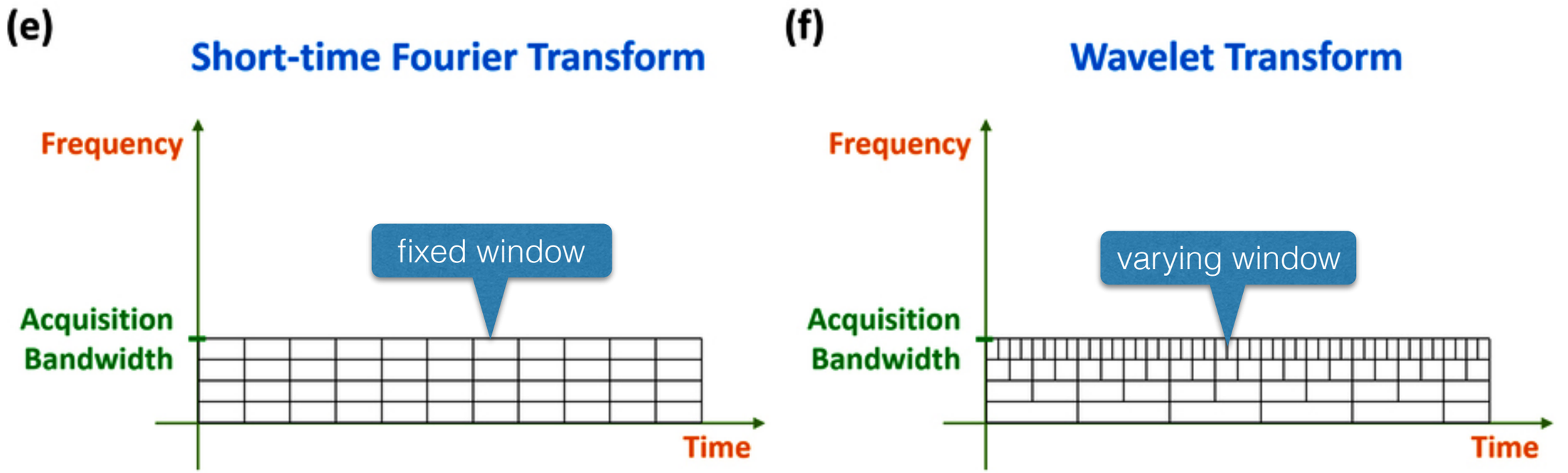
- Multi-resolution nature of wavelet transform
  - Shorter windows of signal are considered for higher frequencies



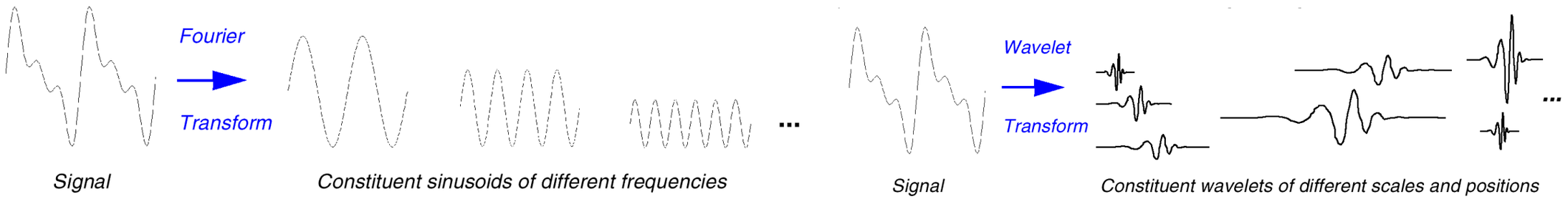
Erol S., Time-Frequency Analyses of Tide-Gauge Sensor Data, Sensors, 2011

Wavelet transform vs. STFT

- STFT uses fixed window size for all frequencies
- Wavelet transform has multi-resolution nature



Mahjoubfar et al., Design of Warped Stretch Transform, Nat. Sci. Rep., 2015



Wavelet transform using MATLAB

- Many wavelet functions in the wavelet toolbox of MATLAB.
- “waveinfo” function lists supported wavelets
  - Continuous wavelet transform (CWT)

    coefs = cwt(x,scales,’wname’)
  - Discrete wavelet transform

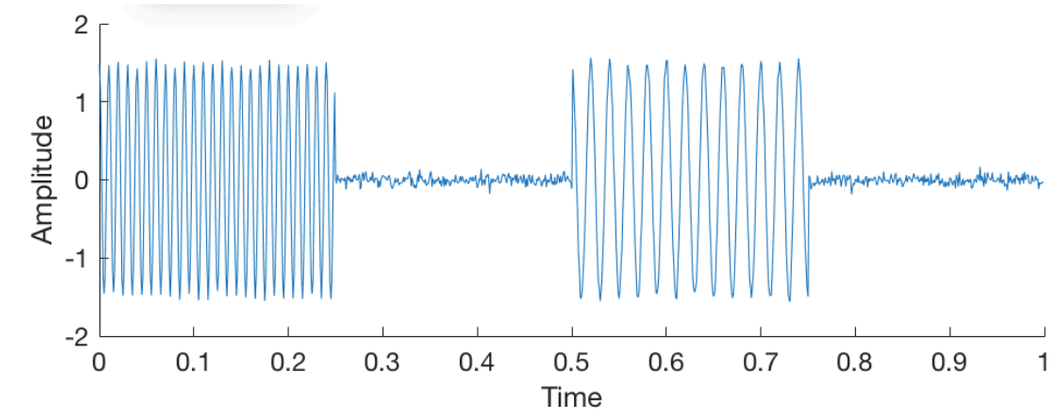
    [C,L] = wavedec(X,N,’wname’)

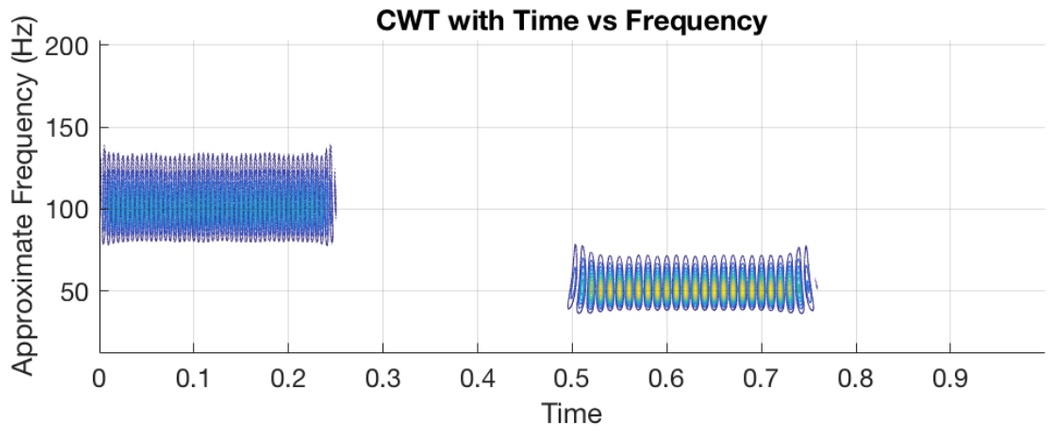
Scale to frequency

- Each scale of wavelet function has a pseudo-frequency
  - an approximate relationship between scale and frequency

F = scal2frq(A,'wname',DELTA)

```
% MATLAB code
Fs = 1000; % sampling frequency of signal
t = 0:1/Fs:1-1/Fs; % time axis
x = 1.5*cos(2*pi*100*t).*(t<0.25)+1.5*cos(2*pi*50*t).*(t>0.5 & t<=0.75);
x = x+0.05*randn(size(t)); % add noise to signal
figure('Color',[1 1 1]);
subplot(211); plot(t, x)
xlabel('Time');
ylabel('Amplitude');
set(gca,'fontsize',12); box off

 numvoices = 32;
a0 = 2^(1/numvoices);
scales = a0.^(2*numvoices:1/numvoices:6*numvoices); % scales
cfs = cwt(x,scales,'morl'); % apply cwt
pfreq = scal2frq(scales,'morl',1/Fs); % match scale to frequency
subplot(212);
contour(t,pfreq,abs(cfs).^2);
axis tight;
grid on;
xlabel('Time');
ylabel('Approximate Frequency (Hz)');
title('CWT with Time vs Frequency');
set(gca,'fontsize',12); box off
```



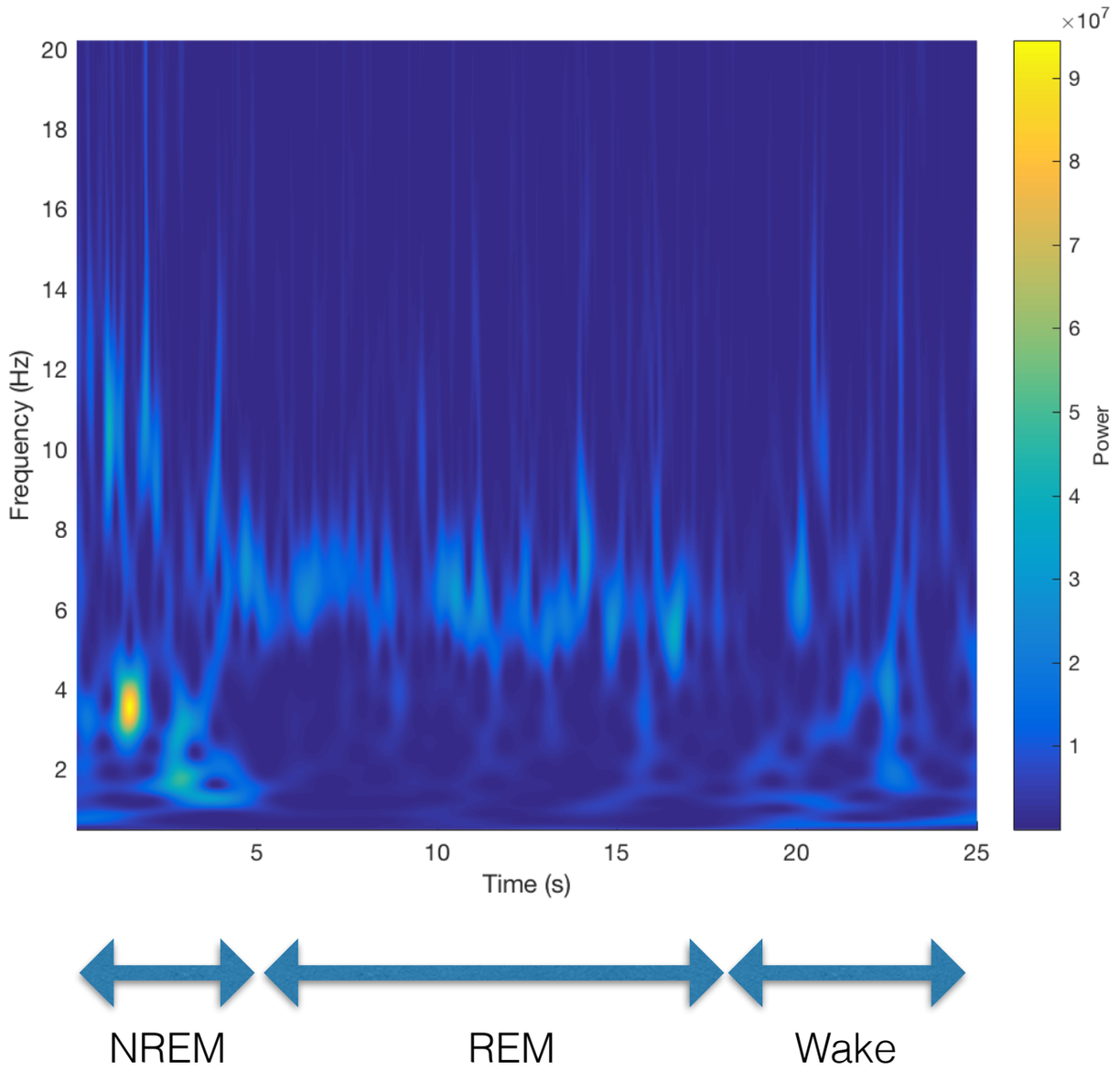


MATLAB help files

Wavelet transform

- Scalogram of hippocampal LFP (NREM, REM, Wake) using Morlet wavelet

```
% MATLAB code
% rawSig and fs are provided in lecture notes)
% rawSig is hippocampal LFP from mouse (fs = 1000 Hz)

% define min/max frequencies (Hz) for analysis
minfreq = 0.5; % min frequency
maxfreq = 20;  % max frequency

% finding corresponding scales for min/max frequencies
dt = 1/fs;   % time between two samples (1/sampling freq.)
f0 = 6/(2*pi); % central freq. of Morlet function ('morl')
NumVoices = 32;
a0 = 2^(1/NumVoices);
minscale = floor(NumVoices*log2(f0/(maxfreq*dt)));
maxscale = ceil(NumVoices*log2(f0/(minfreq*dt)));
scales = a0.^(minscale:maxscale).*dt;

% apply wavelet transform
cwtx = cwtft({rawSig,dt},'wavelet','morl','scales',scales);
```



Conclusion

- Short-time Fourier transform
  - suitable for narrowband signals
    - frequency bands of slow and fast oscillations are close
    - a fixed window length
  - improper for wideband signals
    - shorter duration of higher frequency components (e.g. gamma, ripples)
    - different window lengths for slow and fast oscillations
- Multi-resolution time-frequency representation (e.g. wavelet transform)
  - suitable for both narrowband and wideband signals

Questions?