Frequency / time-frequency domain analysis using MATLAB

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BENESCO Lecture series on signal analysis

by Dr. Mojtaba Bandarabadi
email: mojtaba.bandarabadi@insel.ch

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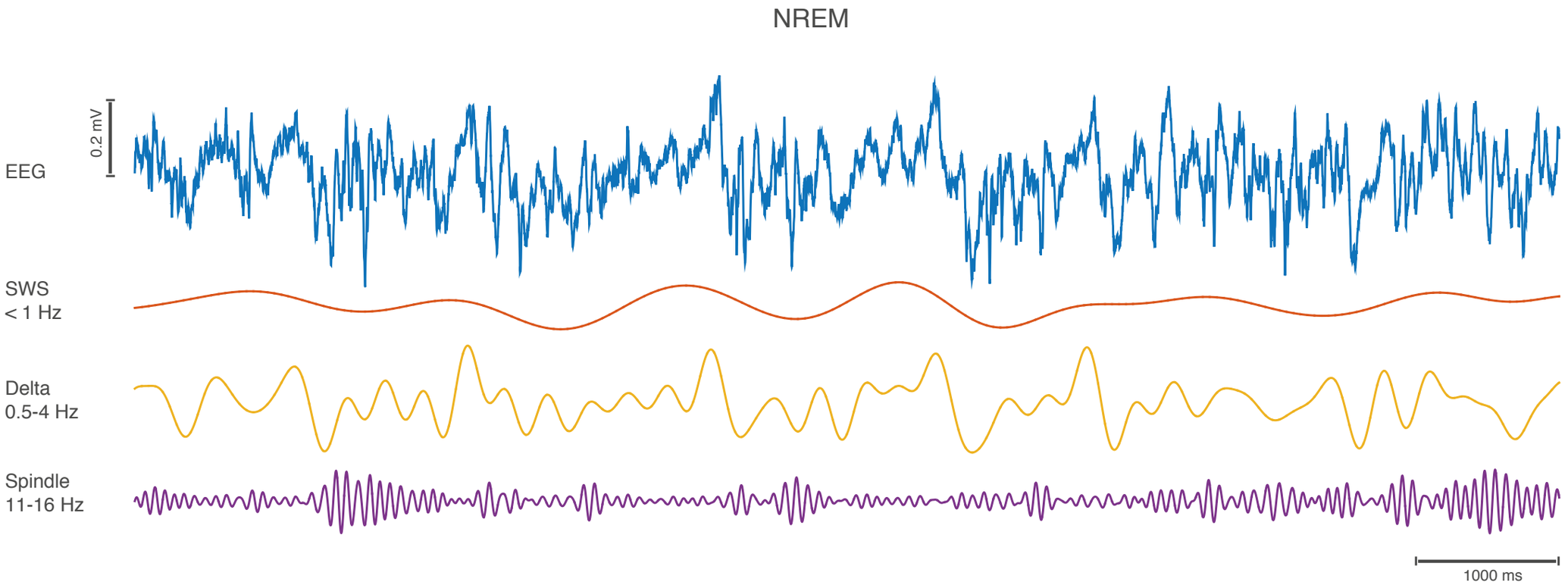
Outline

- Introduction
- Discrete Fourier Transform (DFT)
  - Spectral leakage
  - Windowing
  - Stationarity
- Time-frequency signal analysis
  - Short-time Fourier transform
  - Wavelet analysis
- Spectral analysis of two time series

Introduction

- Signals can be treated in
  - time domain
    - Most signals are function of time
  - frequency domain
    - Mostly distinguished information is hidden in freq. content
  - time-frequency domain
    - time-varying signals, such as brain signals

Frequency domain analysis

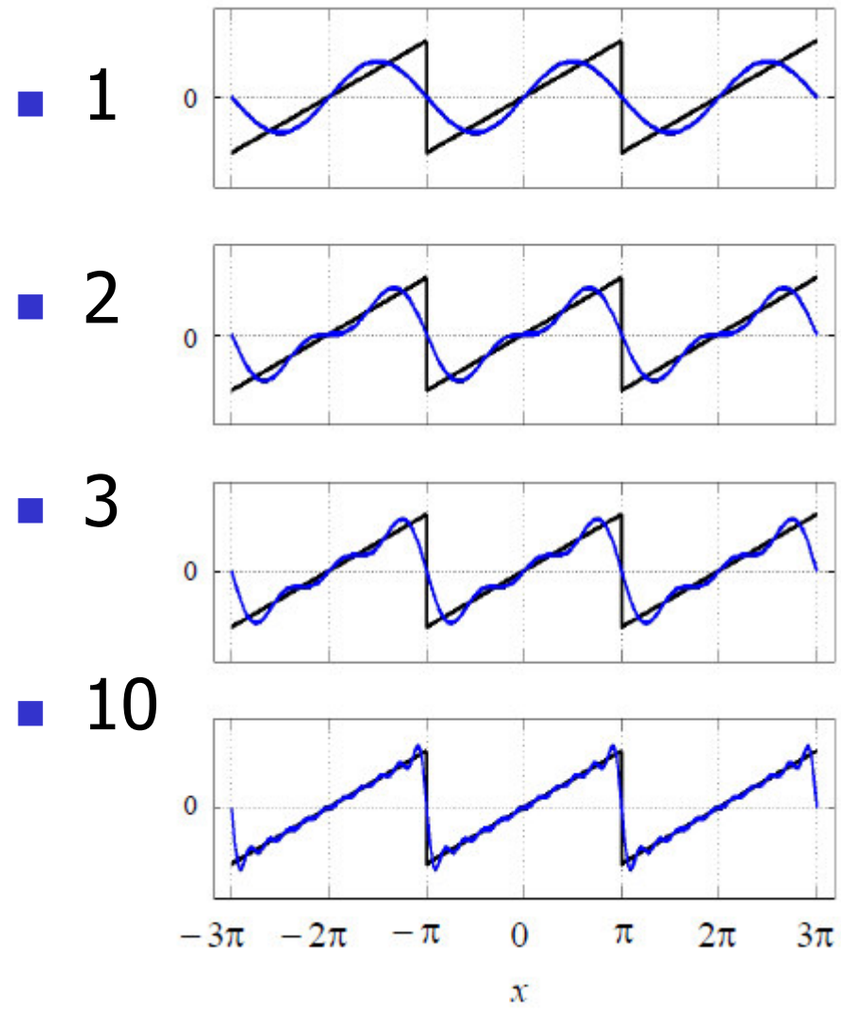


10 s of EEG signal of mouse during non-REM sleep.
Filtered for SWS (Slow Wave Sleep), Delta activity and Spindling

Discrete Fourier Transform (DFT)

- Any waveform can be expressed as a weighted sum of sine/cosine waves!
- DFT provides magnitude and phase at each frequency.

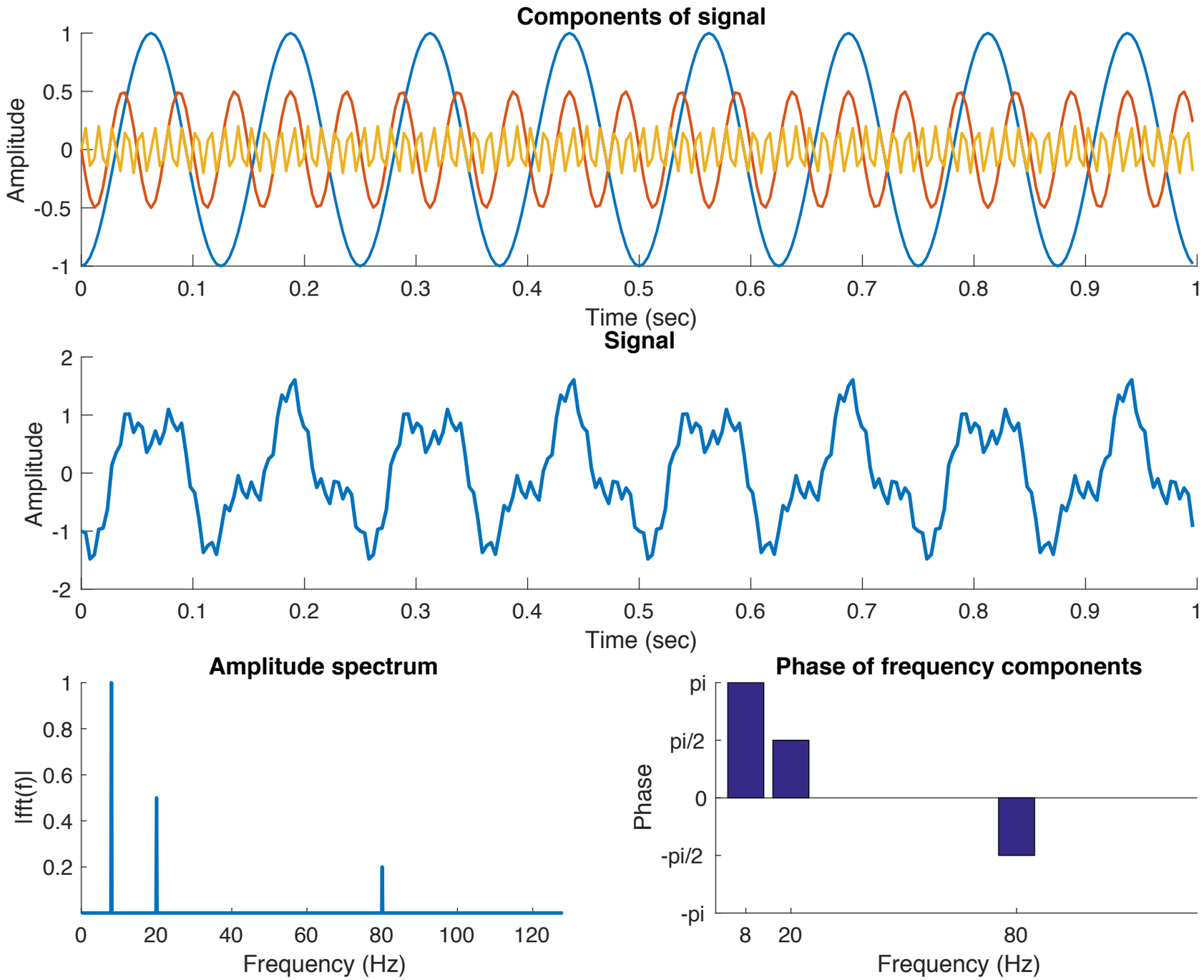
$$X_k \stackrel{\text{def}}{=} \sum_{n=0}^{N-1} x_n \cdot e^{-2\pi i k n / N}, \quad k \in \mathbb{Z}$$



Representing a sawtooth as sum of 10 sine waves.
Source: Wikipedia

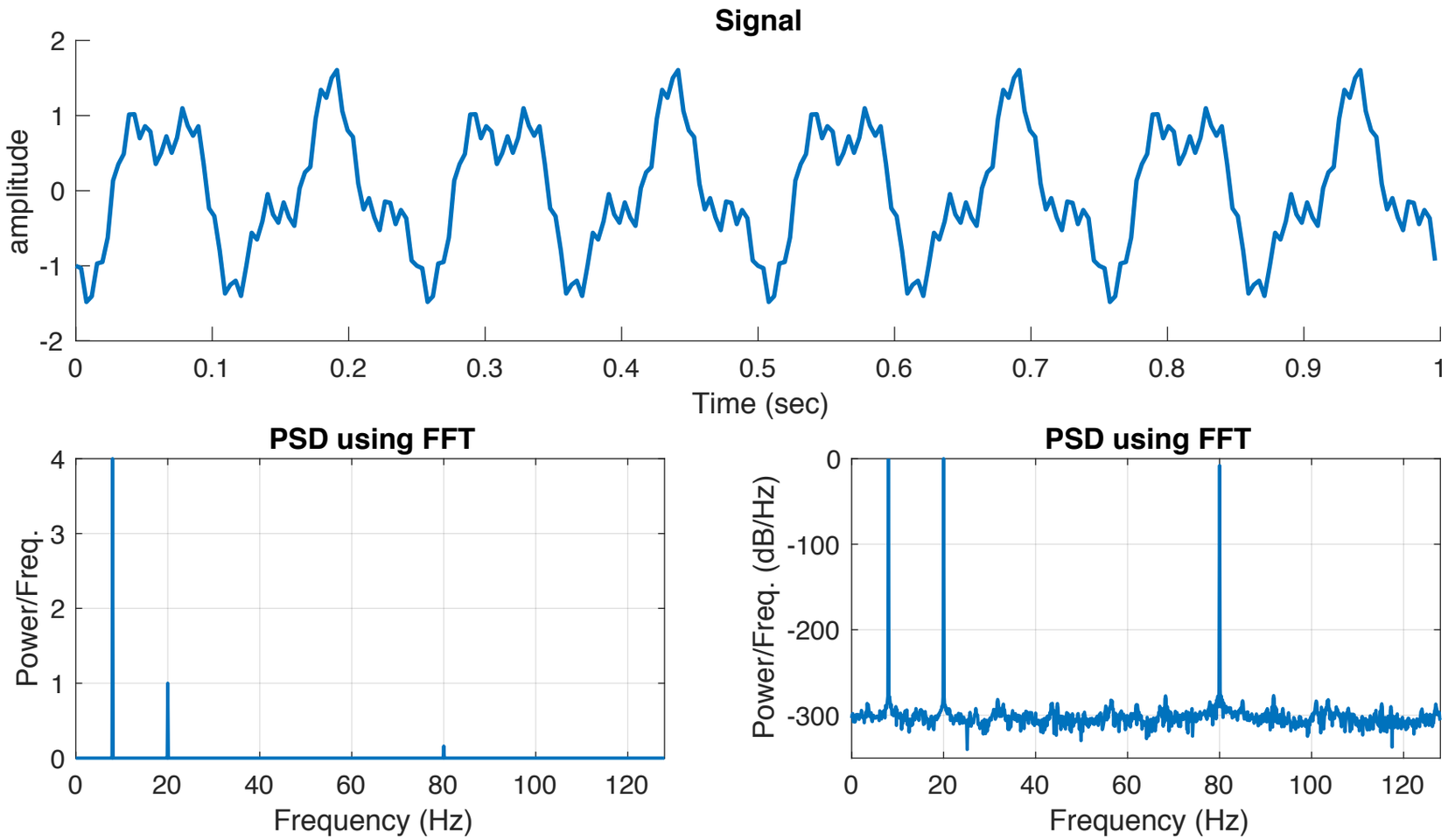
Discrete Fourier Transform (DFT)

```
% MATLAB
% 8 Hz with pi phase
comp1 = 1.0*cos(2*pi*8*time+pi);
% 20 Hz with +pi/2 phase
comp2 = 0.5*cos(2*pi*20*time+pi/2);
% 80 Hz with -pi/2 phase
comp3 = 0.2*cos(2*pi*80*time-pi/2);
signal = comp1 + comp2 + comp3;
```



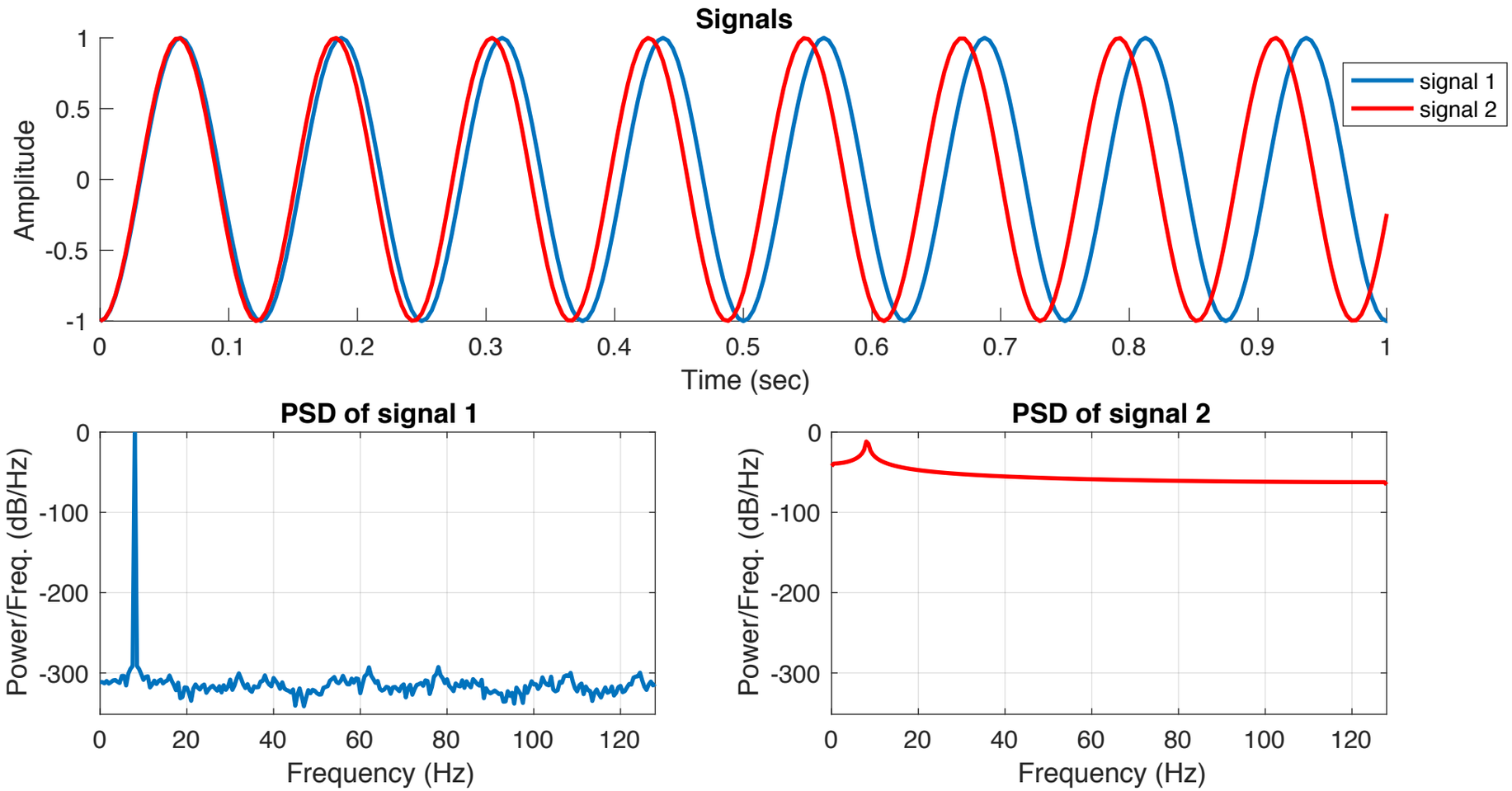
Power spectral density (PSD)

- FFT provides **amplitude** and **phase** of each contributing frequency
- PSD describes how **power** of a signal is distributed over frequency



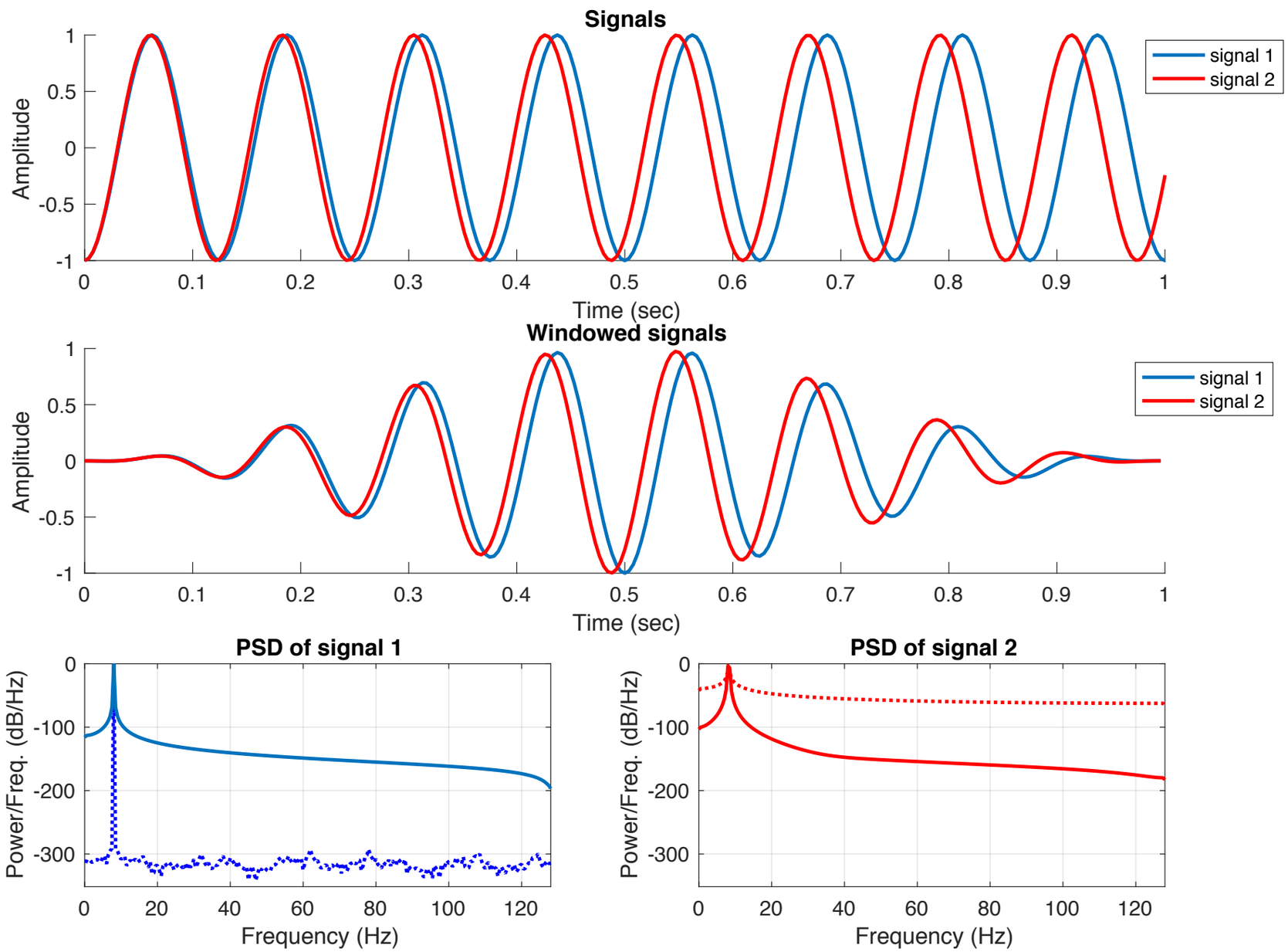
Spectral leakage

- If the number of cycles of a frequency is not an integer (in a sequence of data), the endpoints are discontinuous.
- These artificial discontinuities show up in the FFT as other frequency components not present in the original signal.



Spectral leakage and windowing

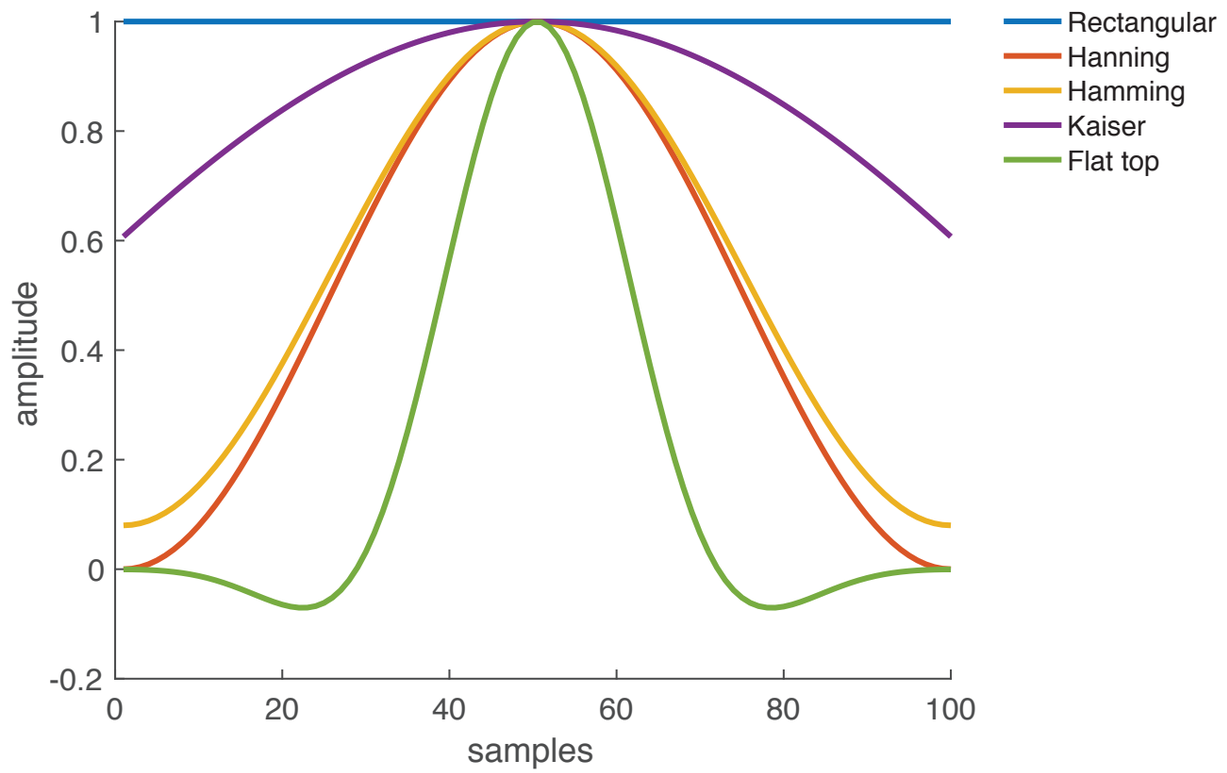
- This is unavoidable for DFT, but can be improved using windowing.



Windowing

- There are several window types for different situations/applications
  - Hanning, Hamming, Kaiser, … (see http://ch.mathworks.com/help/signal/windows.html)
  - In general, the Hanning window is satisfactory in 95 percent of cases.

$$w_j = \frac{1}{2}\left[1 - \cos\left(\frac{2\pi j}{N-1}\right)\right] \quad j \in [0 \ldots N-1]$$

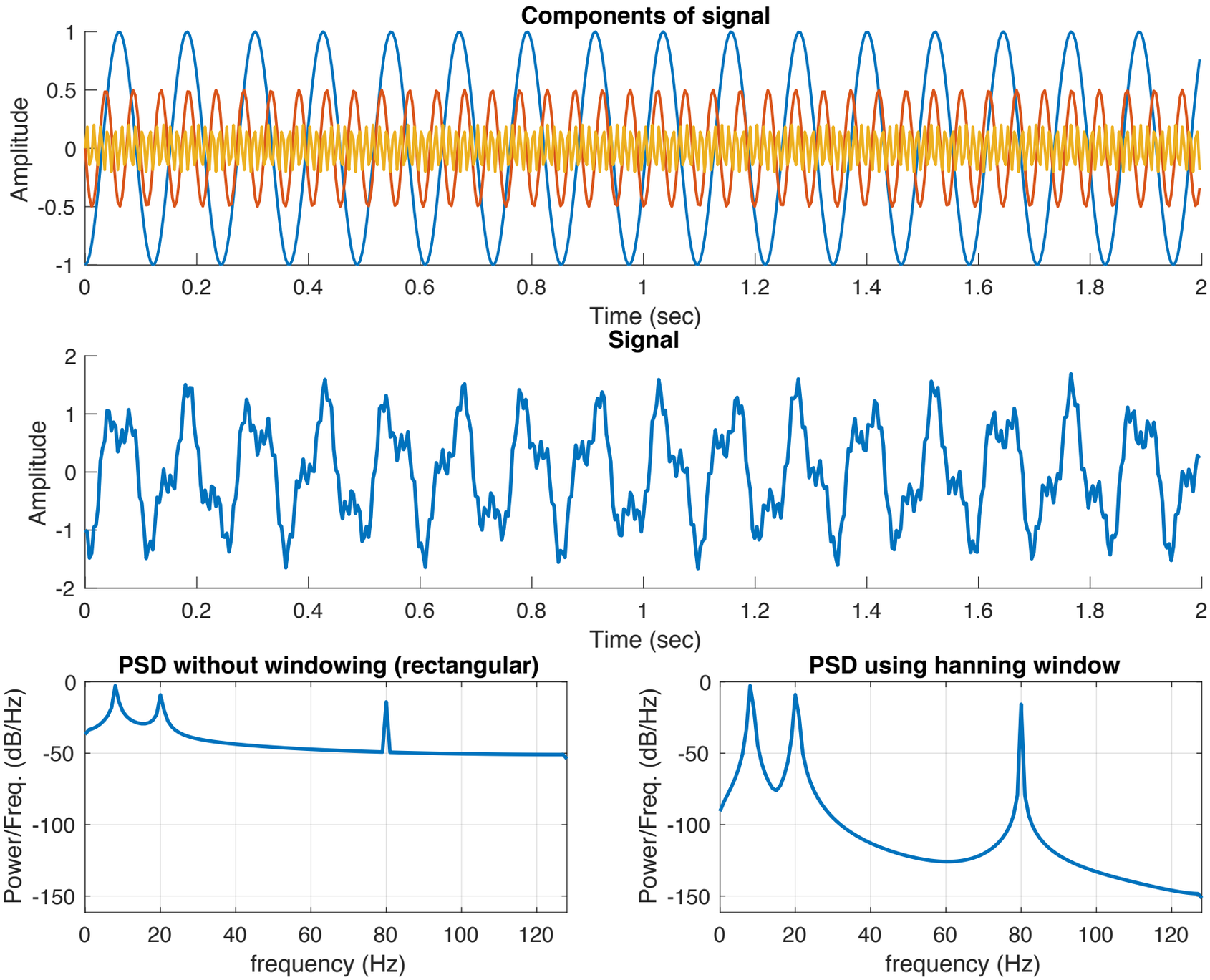


Choice of window

Signal Content	Window
Sine wave or combination of sine waves	Hann
Sine wave (amplitude accuracy is important)	Flat Top
Narrowband random signal (vibration data)	Hann
Broadband random (white noise)	Uniform
Closely spaced sine waves	Uniform, Hamming
Excitation signals (hammer blow)	Force
Response signals	Exponential
Unknown content	Hann
Sine wave or combination of sine waves	Hann
Sine wave (amplitude accuracy is important)	Flat Top
Narrowband random signal (vibration data)	Hann
Broadband random (white noise)	Uniform
Two tones with frequencies close but amplitudes very different	Kaiser-Bessel
Two tones with frequencies close and almost equal amplitudes	Uniform
Accurate single tone amplitude measurements	Flat Top

NATIONAL INSTRUMENTS

Spectral leakage

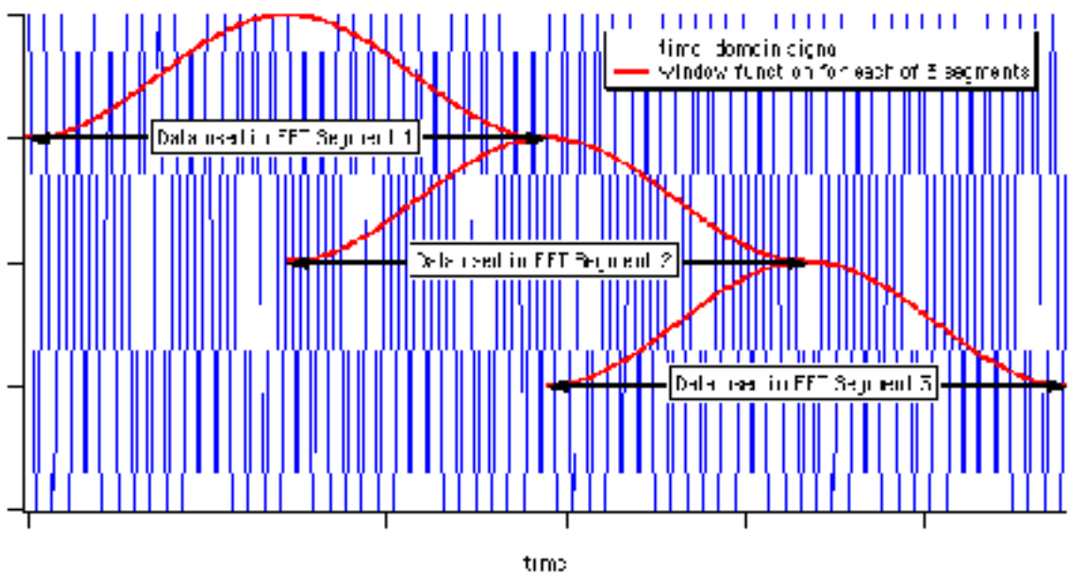


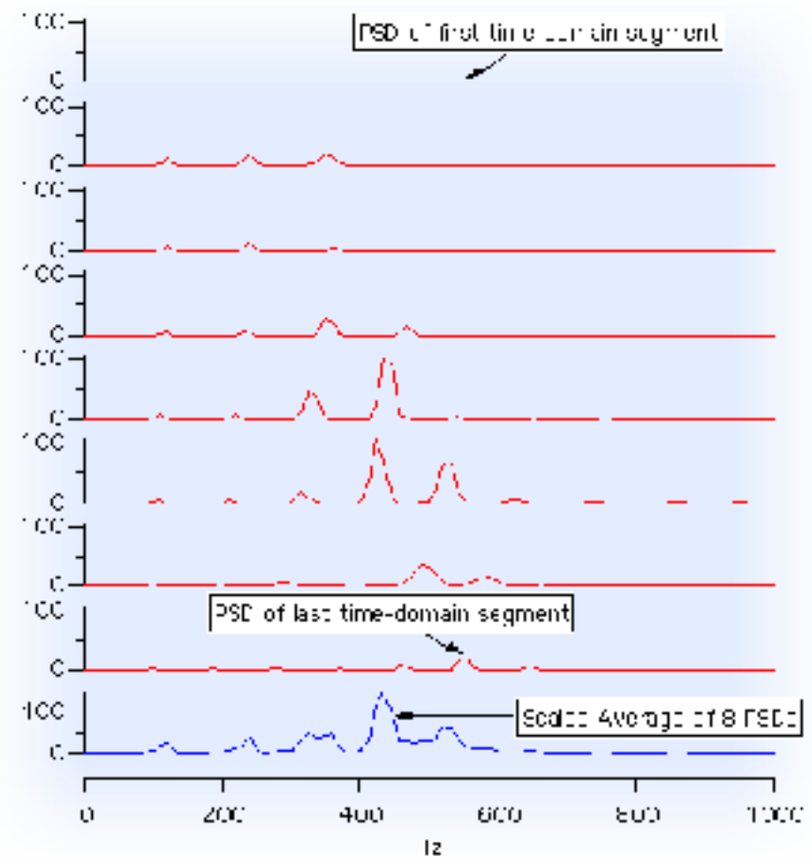
Stationarity

- A stationary time series has constant statistical properties over time, such as mean, variance, and autocorrelation.
  - For stationary signals, frequency domain analysis works well
- Brain is a dynamic complex system
  - Neural signals are non-stationary
    - Frequency content of neural signals changes with time
- How to deal with time-varying signals?
  - Averaging
  - Time-frequency representations (TFRs)

PSD of non-stationary and long signals

- **Welch's method** is an improved PSD estimator, reduces noise by averaging
  - Split up signal into overlapping segments
  - A window function (such as hamming) is applied on segments
  - Squared magnitude of DFT is calculated for each segment
  - Individual PSDs are averaged



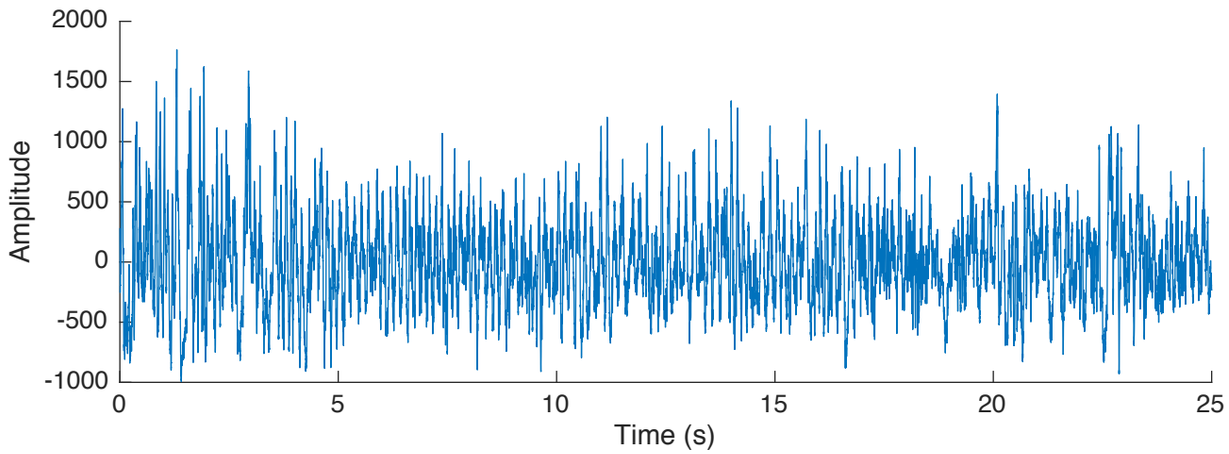


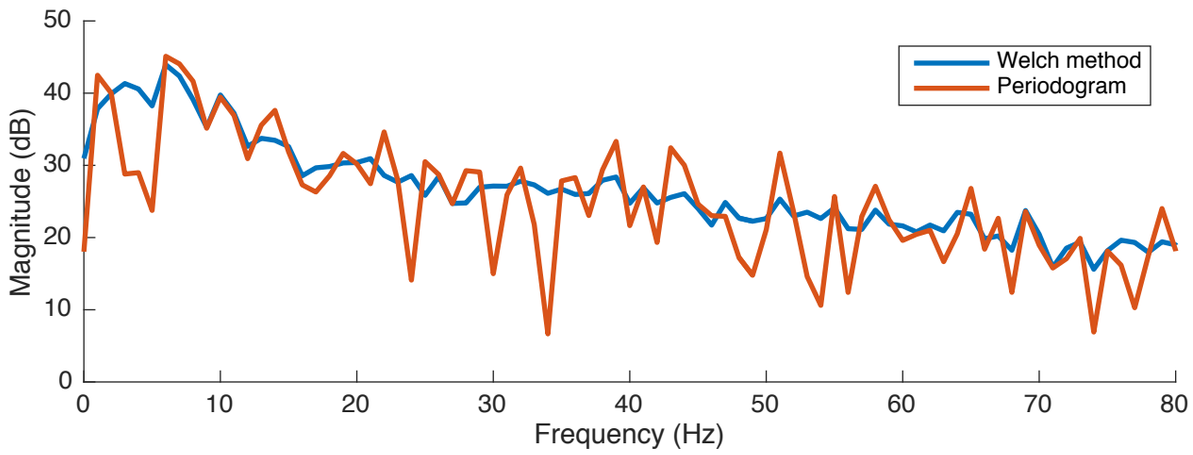
https://www.wavemetrics.com/products/igorpro/dataanalysis/signalprocessing/powerspectra.htm

PSD using Welch's method

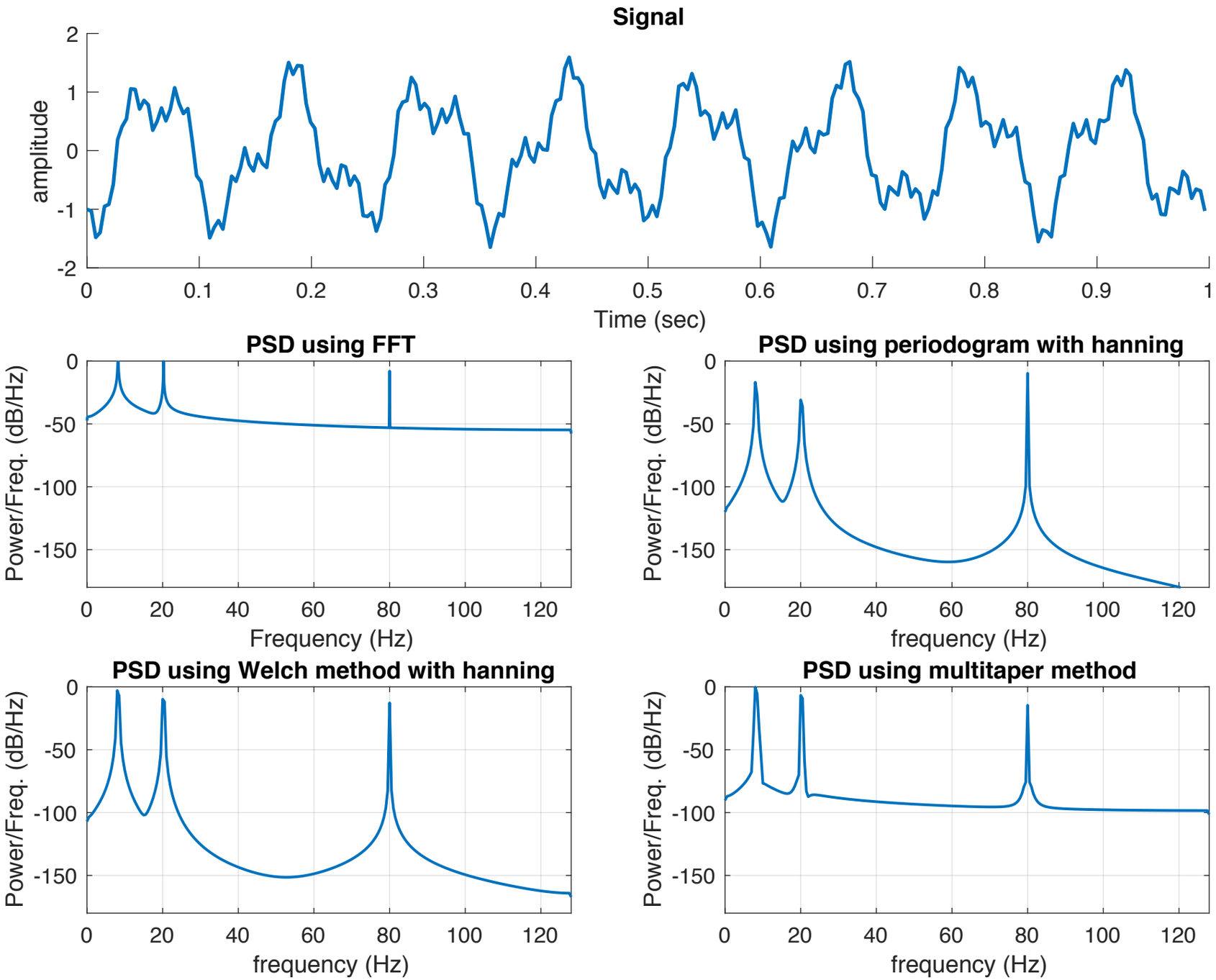
- "pwelch.m" function [pxx,f] = pwelch(x,window,noverlap,f,fs)
- Compare welch's method with periodogram (no windowing)

```
% MATLAB code
% rawSig and fs are provided in lecture notes
% rawSig is hippocampal LFP of mouse (fs = 1000 Hz)
t = 0:1/fs:(length(rawSig)-1)/fs;
subplot(211); plot(t,rawSig); box off
xlabel('Time (s)')
ylabel('Amplitude')

window = 4*fs; % 4-sec windows (min freq 0.25Hz)
overlap = round(0.5*window); % 50% overlap
[pxx,f] = pwelch(rawSig,window,overlap,fs,fs);
subplot(212); hold on
plot(f,10*log10(pxx),'linewidth',2); box off
xlim([0 80])
xlabel('Frequency (Hz)')
ylabel('Magnitude (dB)')

[pxx1,f1] = periodogram(rawSig,...
    rectwin(length(rawSig)),fs,fs);
plot(f1,10*log10(pxx1),'linewidth',2); hold off
xlim([0 80])
xlabel('Frequency (Hz)')
ylabel('Magnitude (dB)')
legend({'Welch method','Periodogram'})
```





PSD - comparing some approaches



What to do with the PSD?

- Finding possible periodicity in the signal
- Quantifying power of desired frequency bands (e.g. delta, theta, alpha, beta, gamma, …, for brain signals)
- Looking for interferences
- Spectral edge frequency and spectral edge power
  - Spectral edge frequency stands for the frequency below which x percent of the overall power of the signal is located.
  - Power covered under spectral edge frequency is called spectral edge power
- …

Conclusion - frequency domain analysis

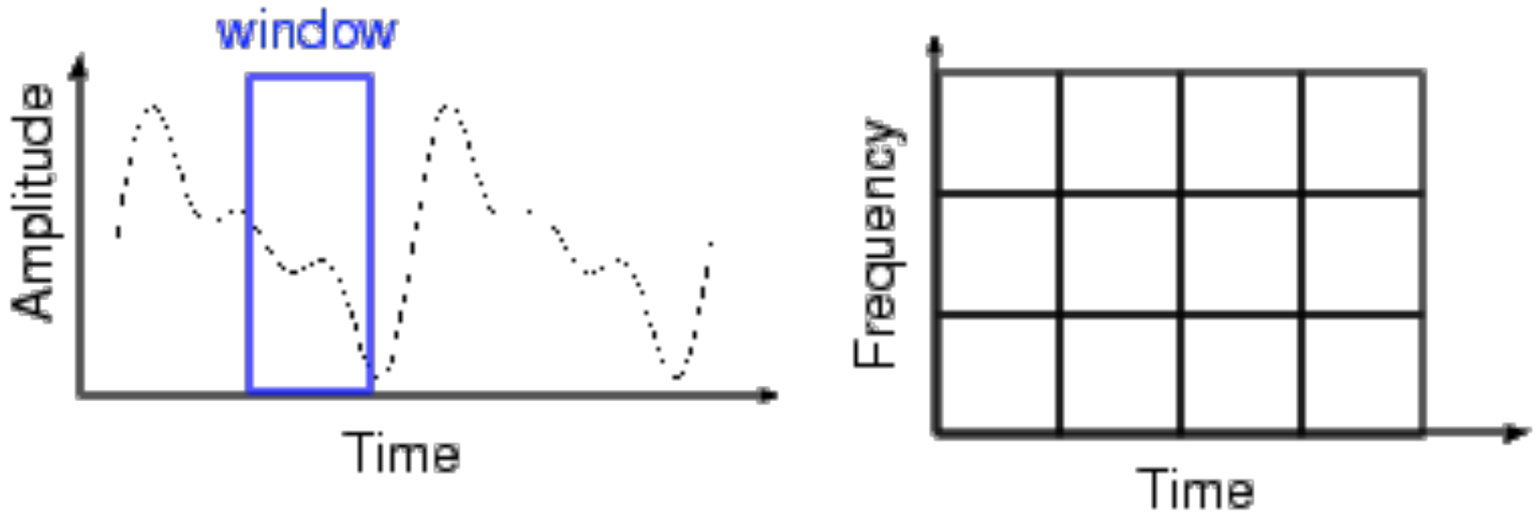
- Useful to have a global view of signal in frequency domain
- Suitable for stationary signals
  - The frequency contents don’t change with time
- Loosing time information
  - When transient changes occur?
  - What frequencies are exist at a particular time?

Time-frequency representation (TFR)

- TFR provides information in time-frequency plane
- Some applications in neural signal processing
  - Optogenetic/electrical stimulation response
  - A visual stimulus
  - Studying of epileptic seizures
  - Analysis of Event-related potential (ERP)
  - Recalling a specific memory
- Some approaches
  - Short-time Fourier transform (spectrogram)
  - Wavelet transform (scalogram), wavelet coherence
  - Wigner distribution
  - Choi-Williams distribution
  - Matching pursuit

Short-Time Fourier Transform (STFT)

- Signals can be assumed quasi-stationary in short times

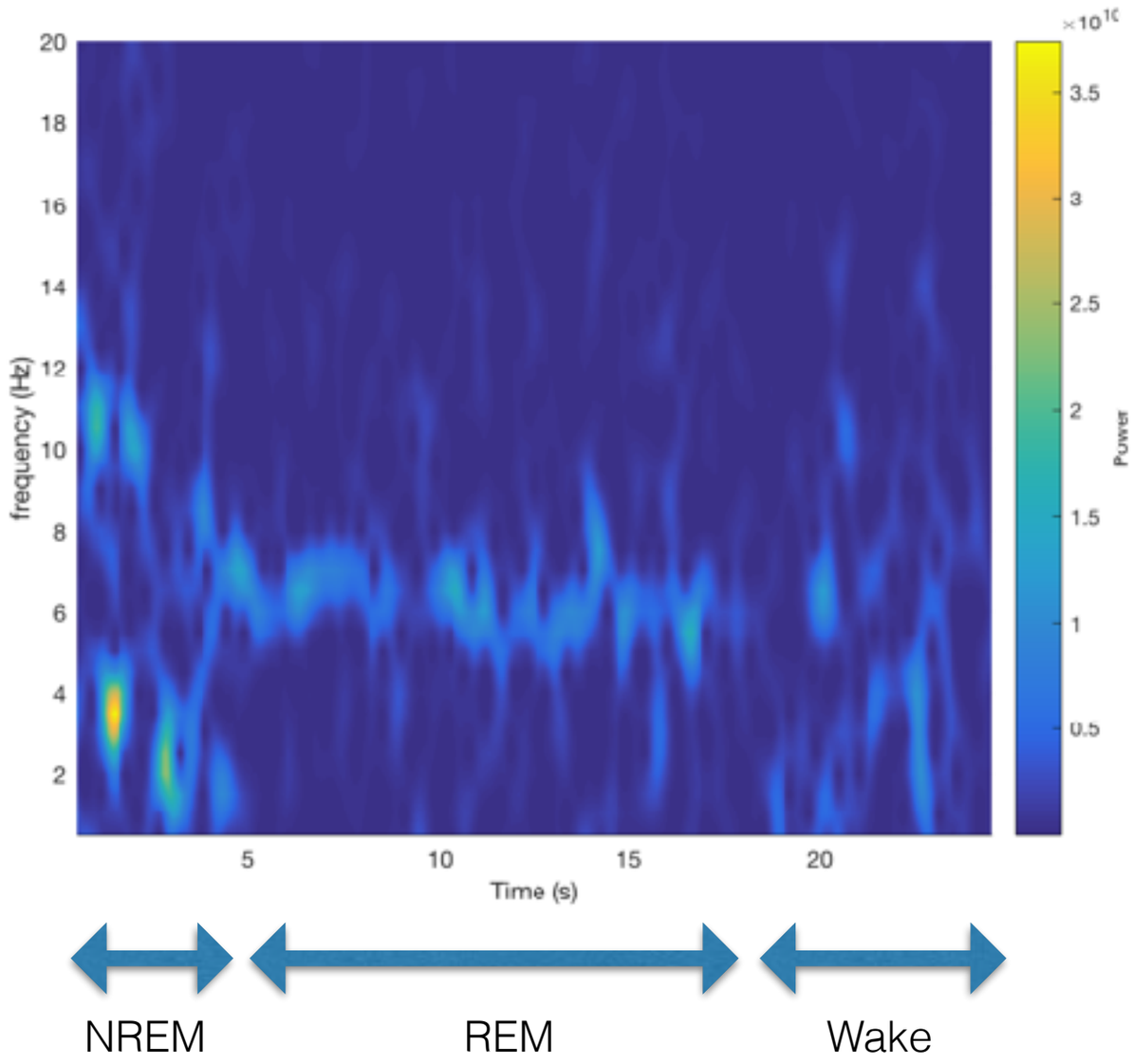


STFT - MATLAB

- “spectrogram.m” function

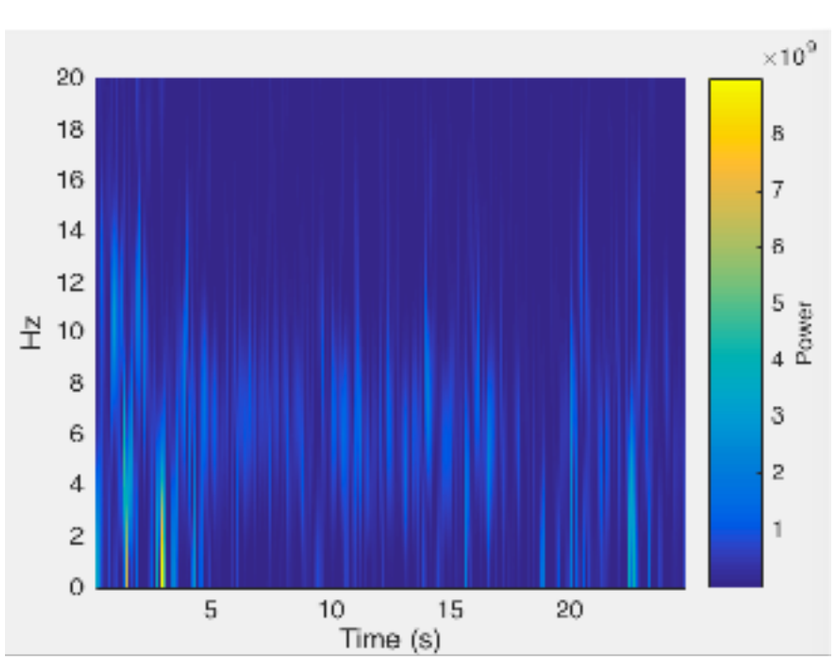
[s,f,t] = spectrogram(x,window,noverlap,f,fs)

```
% MATLAB code
% rawSig and fs are provided in lecture notes
% rawSig is hippocampal LFP from mouse (fs = 1000 Hz)

window = 1*fs; % window size in sample
overlap = round(0.96*window); % overlap is an integer value

[SC,freq,time] = spectrogram(rawSig,window,overlap,2*fs,fs);
SC = abs(SC(freq>=minfreq&freq<=maxfreq,:));
freq = freq(freq>=minfreq&freq<=maxfreq);
args = {time,freq,SC.^2};

figure('Color',[1 1 1]);
surf(args{:},'edgecolor','none');
view(0,90);
axis tight;
shading interp;
colormap(parula(128));
h = colorbar;
h.Label.String = 'Power';
xlabel('Time (s)');
ylabel('frequency (Hz)');
```

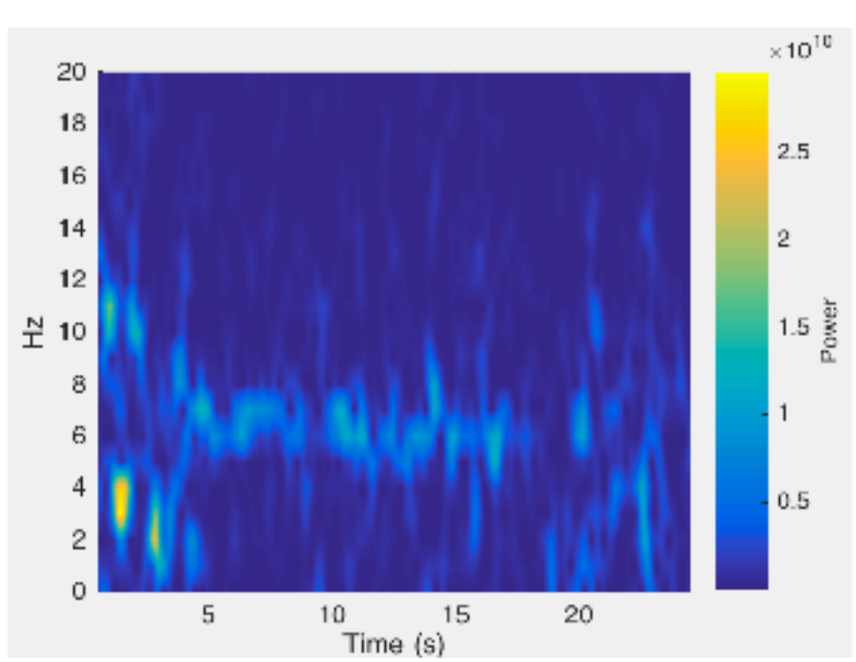


STFT - effect of window width

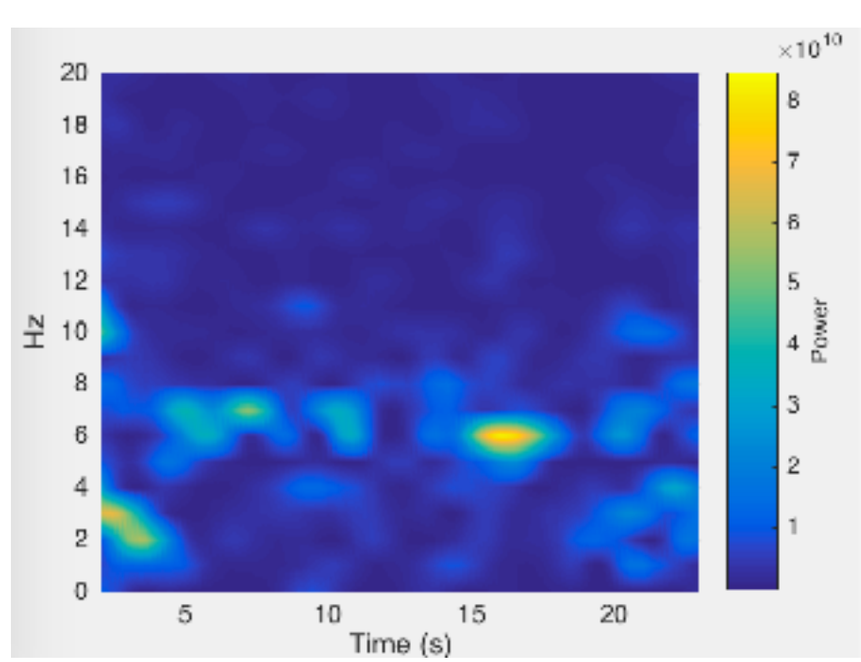
- Control the time and frequency resolutions
  - Narrow window provides better time resolution, degrade freq. resolution
  - Wide window provides better freq. resolution, degrade time resolution



Window length = 0.25 sec



Window length = 1 sec



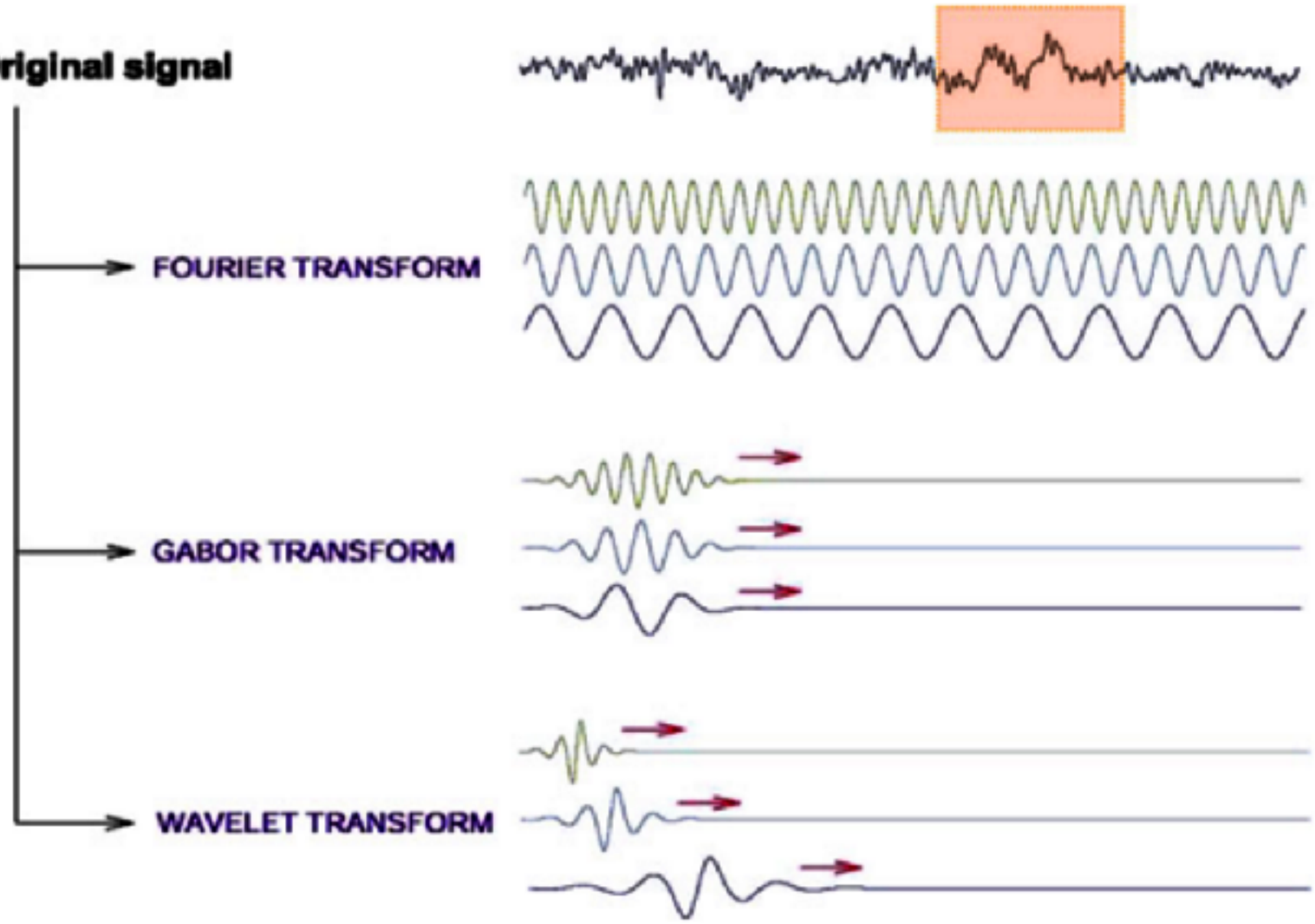
Window length = 4 sec

Conclusion - STFT

- Is useful for signals having narrow frequency changes
  - The range of lower and higher frequency contents is limited.
- Problem in wide-band signals
  - High frequency patterns have a shorter duration in comparison to low frequency patterns.
    - A fixed window for all frequencies (as in STFT) would not work well.
- Solution
  - Multi-resolution time-frequency representation

Wavelet transform

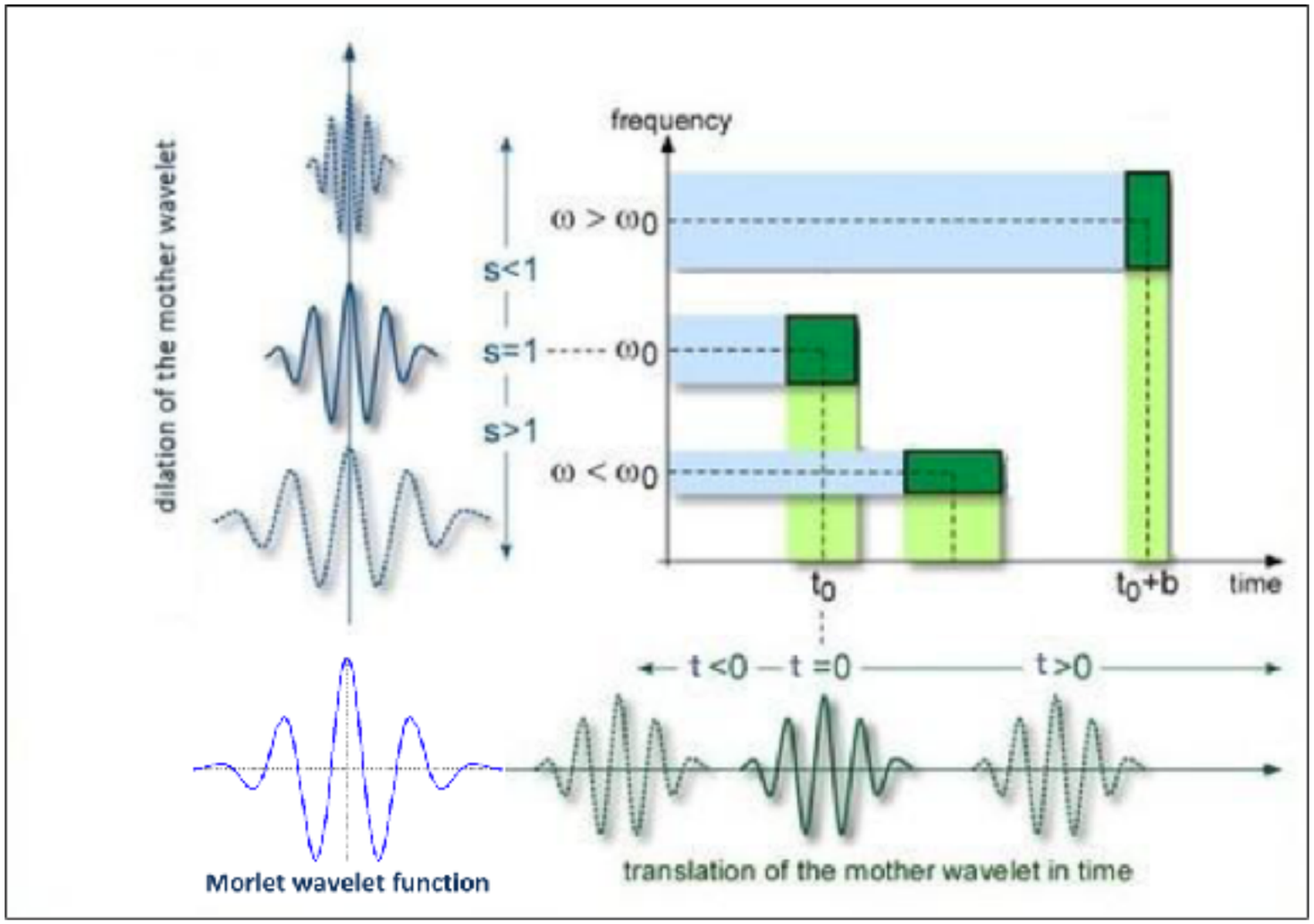
- Convolution between signal and scaled (stretched/shrunk) versions of wavelet function.



Freeman W., Quiroga R. Q., Imaging Brain Function With EEG, Springer, 2013

Wavelet transform

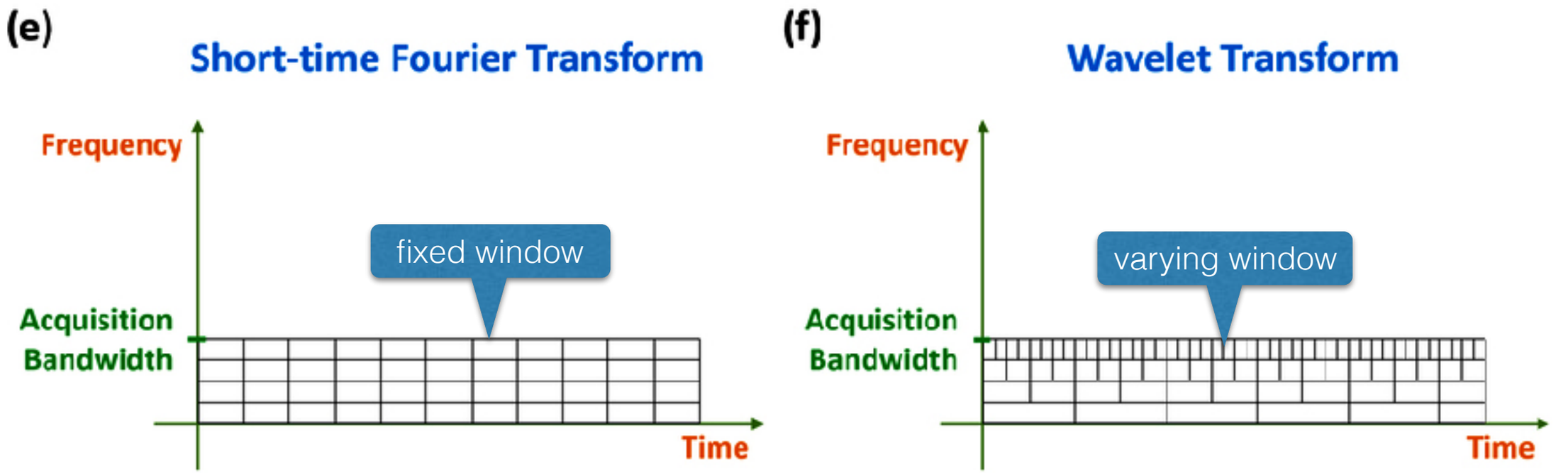
- Multi-resolution nature of wavelet transform
  - Shorter windows of signal are considered for higher frequencies



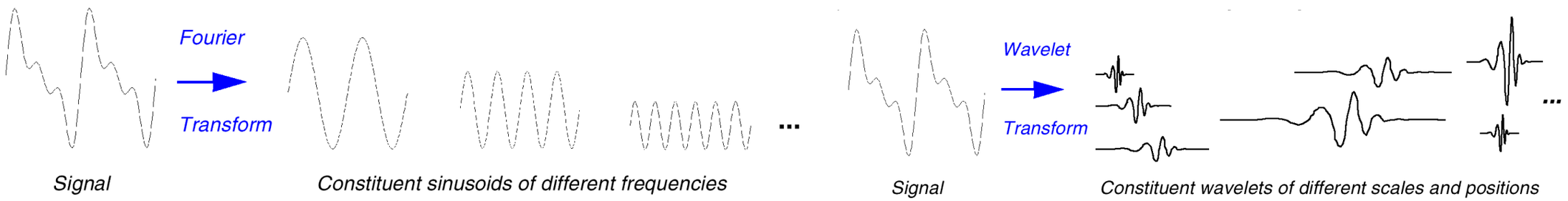
Erol S., Time-Frequency Analyses of Tide-Gauge Sensor Data, Sensors, 2011

Wavelet transform vs. STFT

- STFT uses fixed window size for all frequencies
- Wavelet transform has multi-resolution nature



Mahjoubfar et al., Design of Warped Stretch Transform, Nat. Sci. Rep., 2015



Wavelet transform - MATLAB

- There are many mother wavelets in the MATLAB wavelet toolbox.
- “waveinfo.m” function of MATLAB lists supported wavelets
- Some functions
  - Continuous wavelet transform (CWT)

    coefs = cwt(x,scales,’wname’)
  - CWT using FFT algorithm (faster than cwt function)

    cwtstruct = cwtft(sig,Name,Value)
  - Discreet wavelet transform

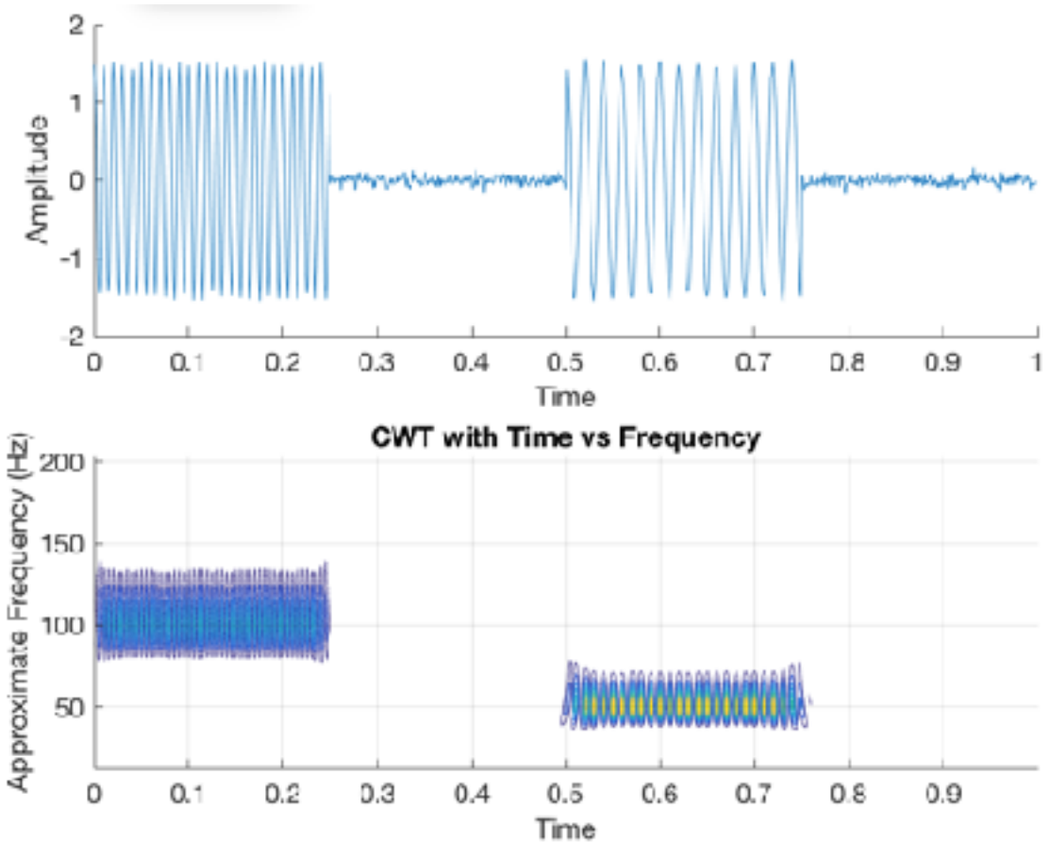
    [C,L] = wavedec(X,N,’wname’)

Scale to frequency

- Each scale of wavelet function has a pseudo-frequency
  - An approximate relationship between scale and frequency)

F = scal2frq(A,'wname',DELTA)

```
% MATLAB code
Fs = 1000; % sampling frequency of signal
t = 0:1/Fs:1-1/Fs; % time axis
x = 1.5*cos(2*pi*100*t).*(t<0.25)+1.5*cos(2*pi*50*t).*(t>0.5 & t<=0.75);
x = x+0.05*randn(size(t)); % add noise to signal

figure('Color',[1 1 1]);
subplot(211); plot(t, x)
xlabel('Time');
ylabel('Amplitude');
set(gca,'fontsize',12); box off

 numvoices = 32;
a0 = 2^(1/numvoices);
scales = a0.^(2*numvoices:1/numvoices:6*numvoices); % scales
cfs = cwt(x,scales,'morl'); % apply cwt
pfreq = scal2frq(scales,'morl',1/Fs); % match scale to frequency
subplot(212);
contour(t,pfreq,abs(cfs).^2);
axis tight;
grid on;
xlabel('Time');
ylabel('Approximate Frequency (Hz)');
title('CWT with Time vs Frequency');
set(gca,'fontsize',12); box off
```

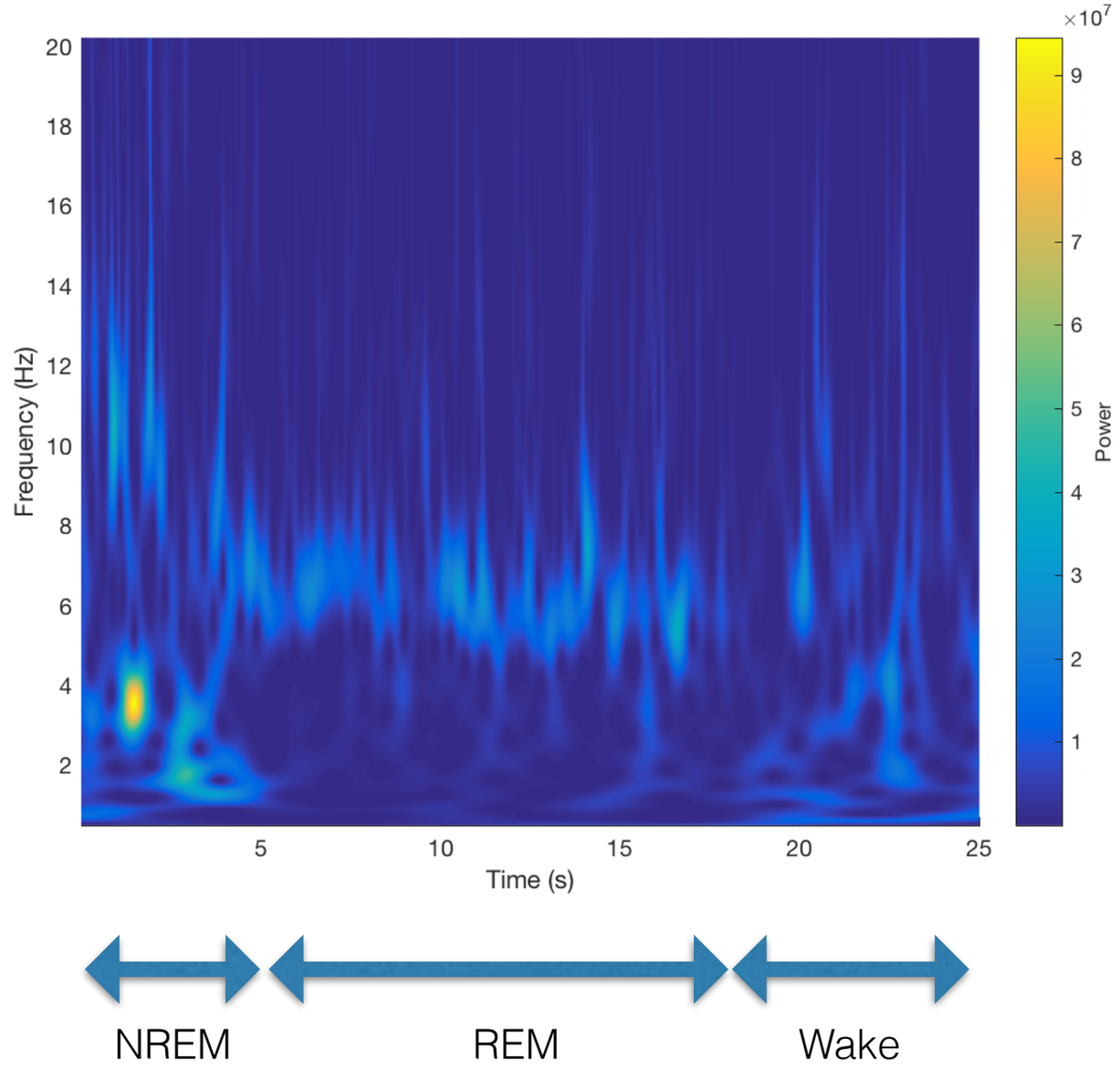


MATLAB help files

Wavelet transform - application

- Scalogram of hippocampal LFP recorded from mouse (NREM, REM, Wake) using Morlet wavelet

```
% MATLAB code
% rawSig and fs are provided in lecture notes)
% rawSig is hippocampal LFP from mouse (fs = 1000 Hz)
minfreq = 0.5; % minimum frequency for CWT
maxfreq = 20;  % maximum frequency for CWT

dt = 1/fs;          % time between two samples (1/sampling freq.)
time = dt:dt:length(rawSig)/fs; % define time axis
f0 = 6/(2*pi);      % central frequency of wavelet function ('morl')
NumVoices = 32;
a0 = 2^(1/NumVoices);
minscale = f0/(maxfreq*dt);
maxscale = f0/(minfreq*dt);
minscale = floor(NumVoices*log2(minscale));
maxscale = ceil(NumVoices*log2(maxscale));
scales = a0.^(minscale:maxscale).*dt;
cwtx = cwtft({rawSig,dt},'wavelet','morl','scales',scales); % CWT
SC = abs(cwtx.cfs);
freq = cwtx.frequencies;
args = {time,freq,SC.^2};

figure('Color',[1 1 1]);
surf(time,cwtx.frequencies,abs(cwtx.cfs).^2,'edgecolor','none');
view(0,90);
axis tight;
shading interp;
colormap(parula(128));
h = colorbar;
h.Label.String = 'Power';
xlabel('Time (s)');
ylabel('Frequency (Hz)');
```



Frequency analysis of two signals

- Cross power spectral density (CPSD) Provides information on
  - the power shared by a given frequency for the two signals
  - the phase shift information between the two signals (time lag).
- Is obtained using PSD of cross-correlation of two signals

$$P_{xy}(\omega) = \sum_{m=-\infty}^{\infty} R_{xy}(m) e^{-j\omega m}.$$

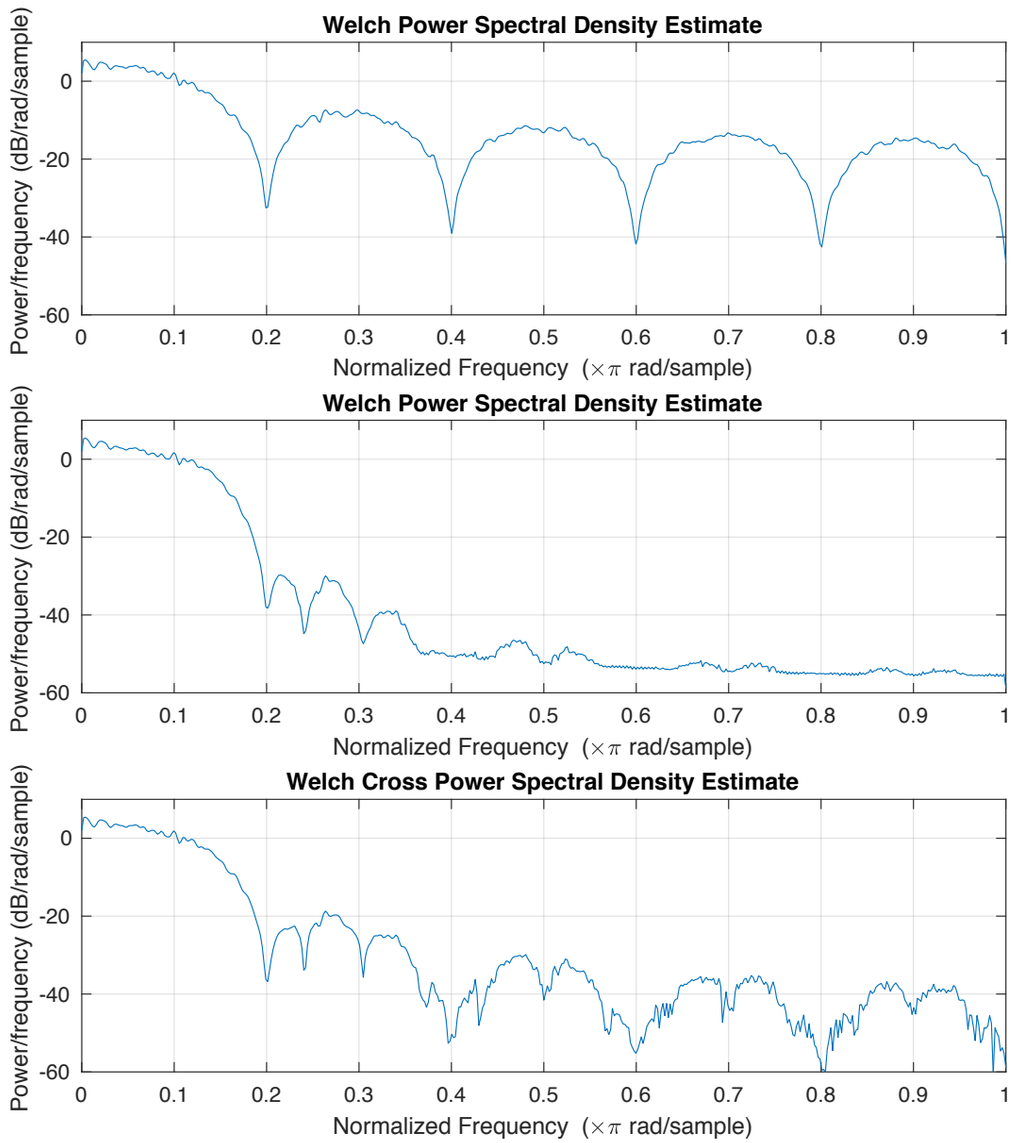
Cross-correlation → $R_{xy}(m) = E\left\{x_{n+m} y_n^*\right\} = E\left\{x_n y_{n-m}^*\right\}$

Cross power spectral density

- “cpsd.m” function

[Pxy,F] = cpsd(x,y,window,noverlap,nfft,fs)

```
% MATLAB code
h1 = ones(1,10)/sqrt(10);
h2 = fir1(30,0.2,rectwin(31));
r = randn(16384,1);
x = filter(h1,1,r);
y = filter(h2,1,x);

figure('Color',[1 1 1]);
subplot(311); pwelch(x,500,250,1024); ylim([-60 10])
subplot(312); pwelch(y,500,250,1024); ylim([-60 10])
subplot(313); cpsd(x,y,500,250,1024); ylim([-60 10])
```



MATLAB help files

Time-frequency analysis of two signals

- The cross examination of the two CWT decompositions obtained from two signals
  - can reveal localized similarities in time-scale plane.
  - can reveals common time-varying patterns
  - Wavelet cross spectrum and coherence are two measures

Wavelet cross spectrum

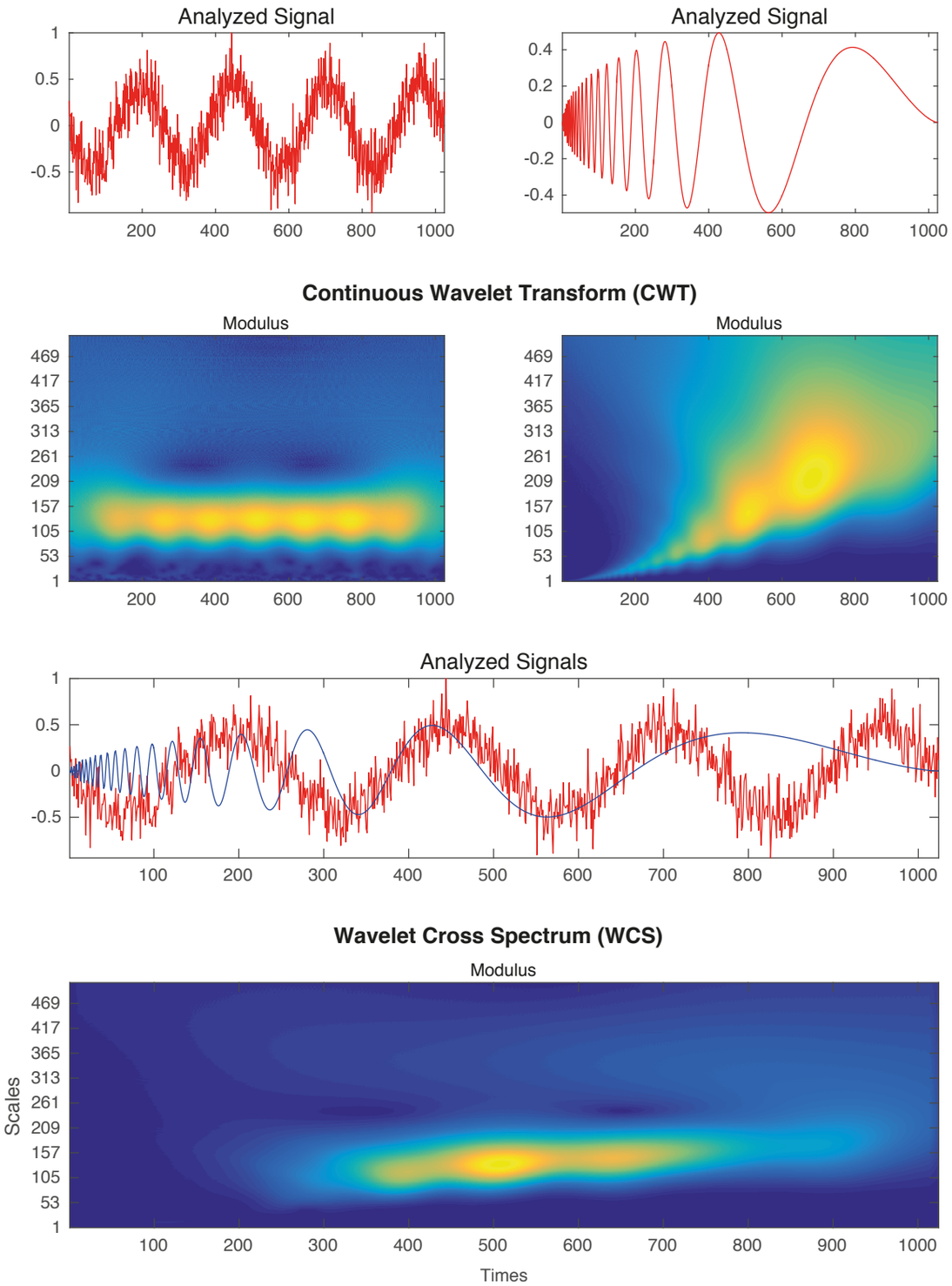
- The cross spectrum of two signals (x,y) denotes the fraction of covariance at each scale a and time b.

$$C_{xy}(a,b) = S(C_x^*(a,b)C_y(a,b))$$

- The phase spectrum denotes the phase difference between the signals at each scale a and time b.

Wavelet cross spectrum - MATLAB

```
% MATLAB code

wname  = 'cgau2'; % wavelet function
scales = 1:512;   % scales to plot
ntw = 21;         % length of moving average

t = linspace(0,1,1024); % time
x = -sin(8*pi*t) + 0.4*randn(1,1024);
x = x/max(abs(x));        % first signal
y = wnoise('doppler',10); % second signal

wcoher(x,y,scales,wname,'ntw',ntw,'plot','cwt');

figure;
wcoher(x,y,scales,wname,'ntw',ntw,'nsw',1,'plot','wcs');
```



MATLAB help files

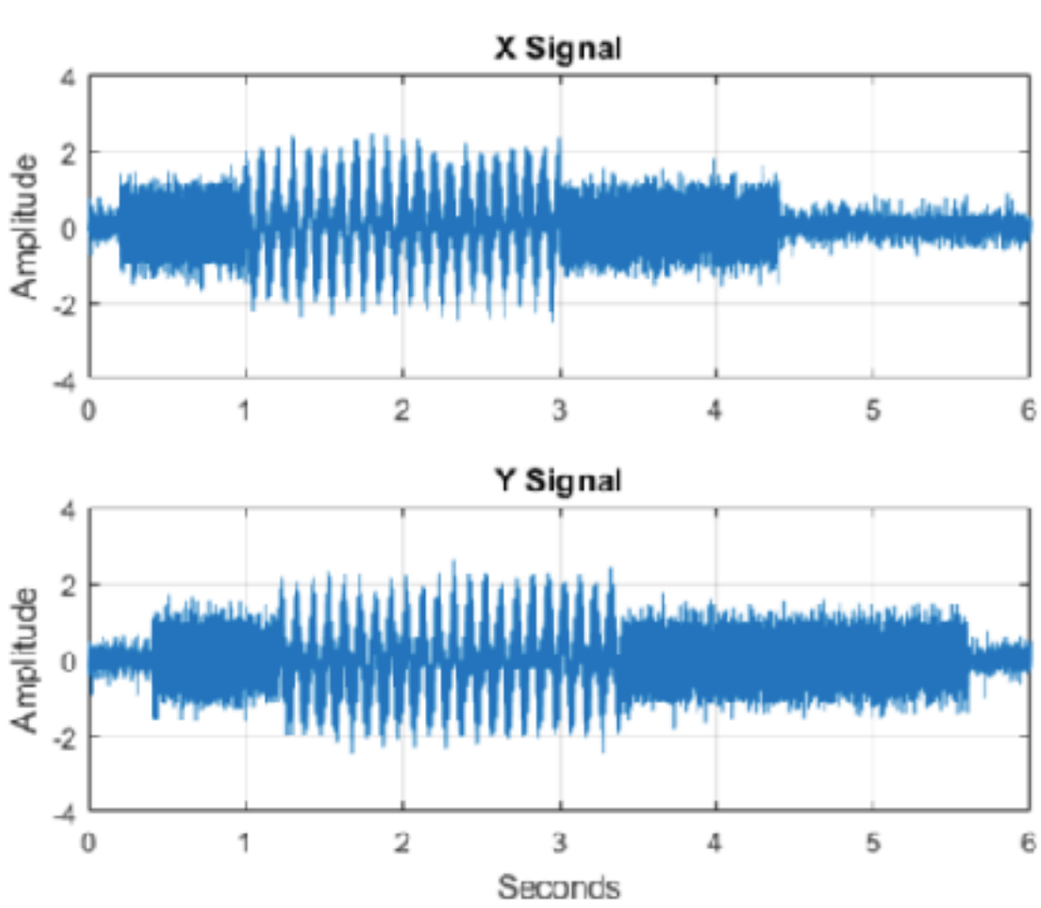
Wavelet coherence

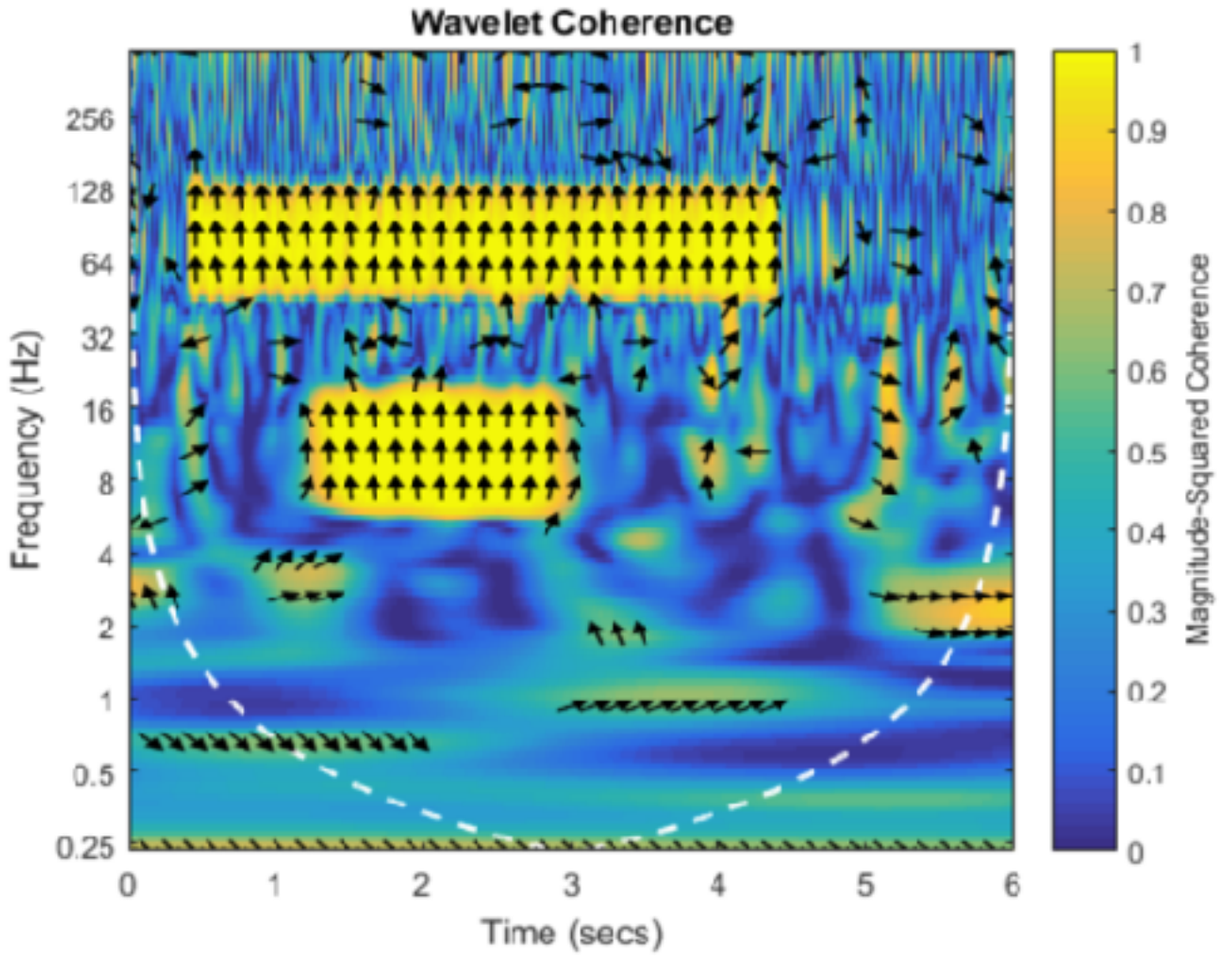
- Time-varying coherence is a powerful tool for revealing functional dynamics between different regions in the brain.
- Is the normalized cross-spectrum with respect to the spectrum of each signal (a value between 0 and 1)

$$\frac{\left|S(C_x^*(a,b)C_y(a,b))\right|^2}{S(\left|C_x(a,b)\right|^2 S(\left|C_y(a,b)\right|^2}$$

Wavelet coherence

- Measure the degree of a linear relationship between the two signals at different scales and times.
- “wcoher.m” function of MATLAB provides





MATLAB help files

Questions?