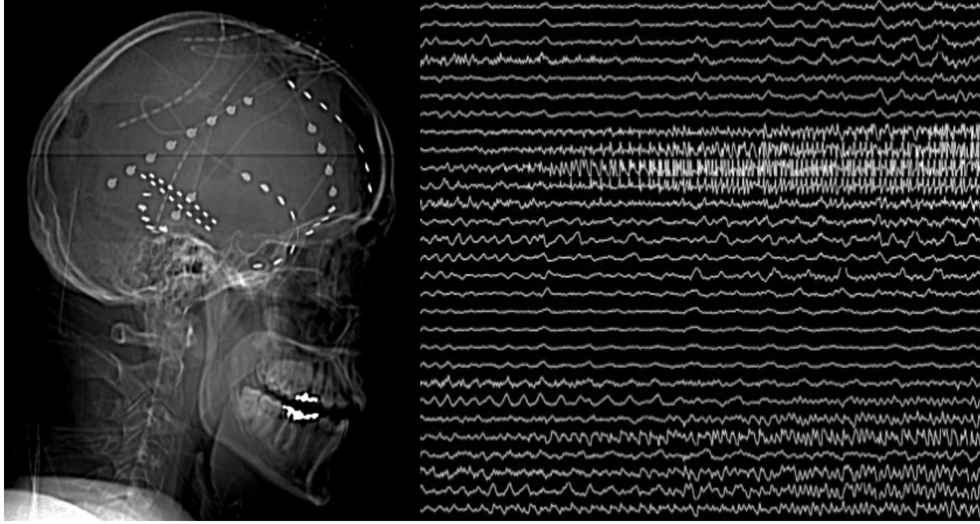
BENESCO Lecture Series
„Signal Analysis and
Brain Oscillations in Health and Disease“

Bern, April 1st 2016



Correlation analysis of multivariate time series and Principle Component Analysis
– another lecture without (too many) formulae

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it could be as simple as that ...

Definitions

$$\mathbf{\Sigma} = \frac{1}{T}(\mathbf{X} - \boldsymbol{\mu}_X)(\mathbf{X} - \boldsymbol{\mu}_X)' \tag{1}$$

$$\mathbf{\Sigma}\,\mathbf{v}_k = \lambda_k\,\mathbf{v}_k \quad \text{with} \quad \mathbf{v}_l'\mathbf{v}_k = \delta_{lk} \quad \forall k, l \in [1, M] \tag{2}$$

Proposition (spectral decomposition)

$$\mathbf{\Sigma} = \sum_{l=1}^{M} \lambda_l\,\mathbf{v}_l\mathbf{v}_l' \tag{3}$$

Proof

$$\lambda_k\,\mathbf{v}_k = \sum_{l=1}^{M} \lambda_l\,\mathbf{v}_l\,\delta_{lk} = \sum_{l=1}^{M} \lambda_l\,\mathbf{v}_l\,\mathbf{v}_l'\mathbf{v}_k = \mathbf{\Sigma}\,\mathbf{v}_k \qquad \square \tag{4}$$

it could be as simple as that ...

Lemma (principal components)

$$\begin{aligned} \mathbf{V} &= (\mathbf{v}_1 | \ldots | \mathbf{v}_M) \\ \mathbf{Y} &= \mathbf{V}'(\mathbf{X} - \boldsymbol{\mu}_X) \\ \frac{1}{T}\mathbf{Y}\mathbf{Y}' &= \mathrm{diag}(\lambda_1, \ldots, \lambda_M) \end{aligned}$$

Proof

$$\frac{1}{T}\mathbf{Y}\mathbf{Y}' = \frac{1}{T}\mathbf{V}'(\mathbf{X} - \boldsymbol{\mu}_X)(\mathbf{X} - \boldsymbol{\mu}_X)'\mathbf{V} = \mathbf{V}'\boldsymbol{\Sigma}\mathbf{V} = \mathrm{diag}(\lambda_1, \ldots, \lambda_M) \qquad \square$$

it could be as simple as that ...

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… but unfortunately we are no mathematicians

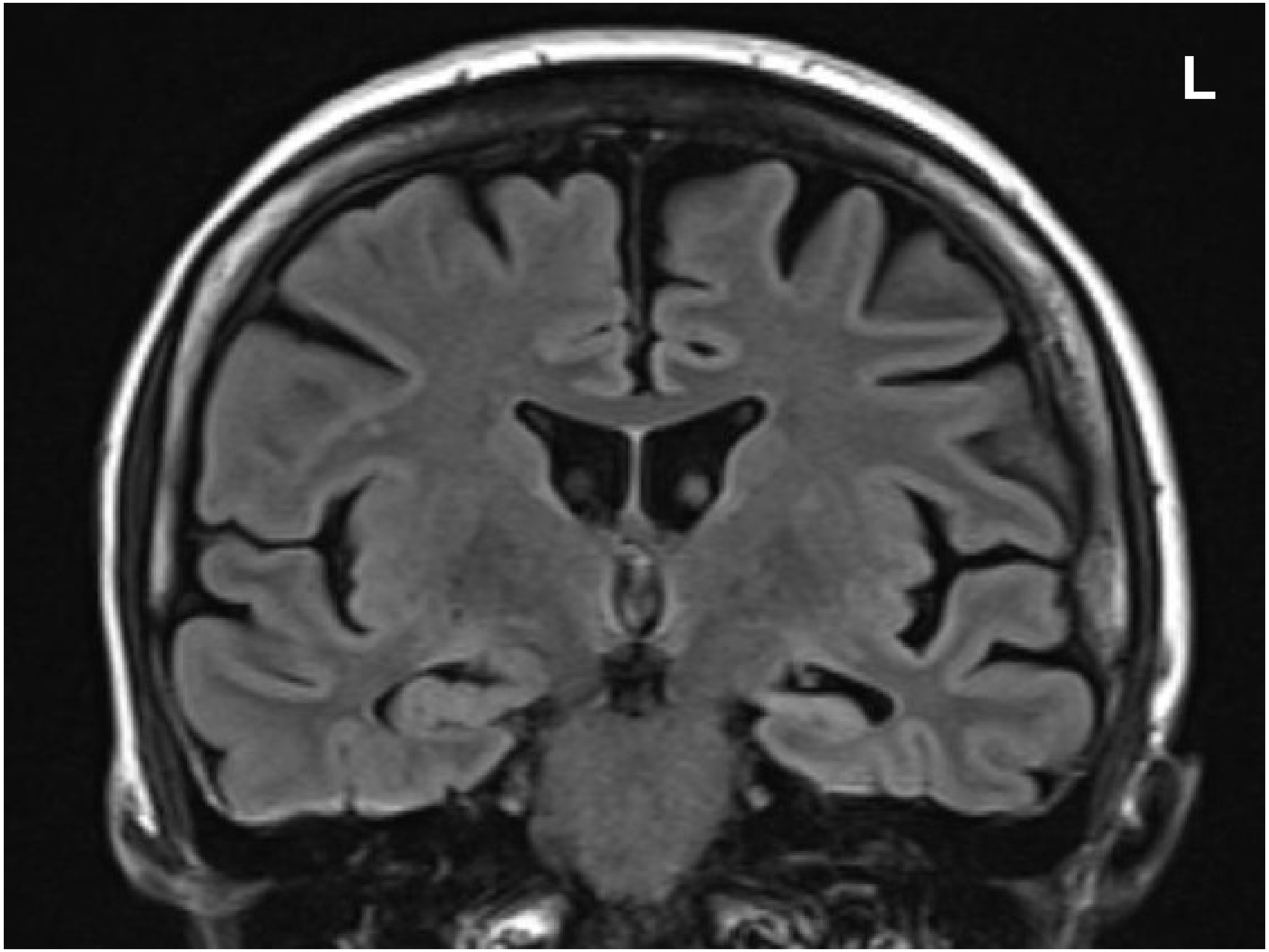
Example EEG

phase II monitoring

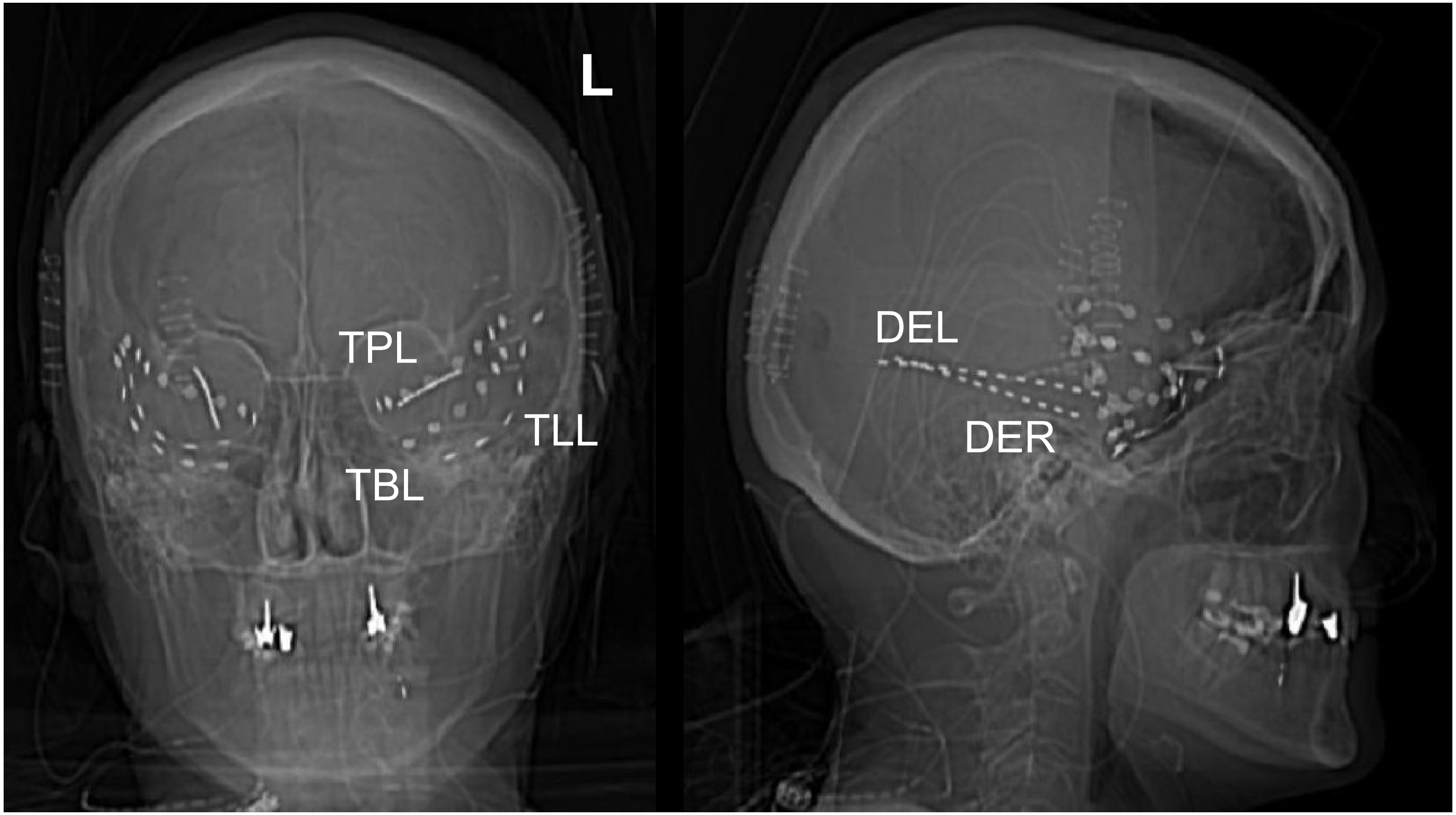
- 48 year old, female
- mesial temporal lobe epilepsy
- hippocampal sclerosis on the left

- 2 depth electrodes
- 6 strip electrodes
- 64 contacts

Example EEG

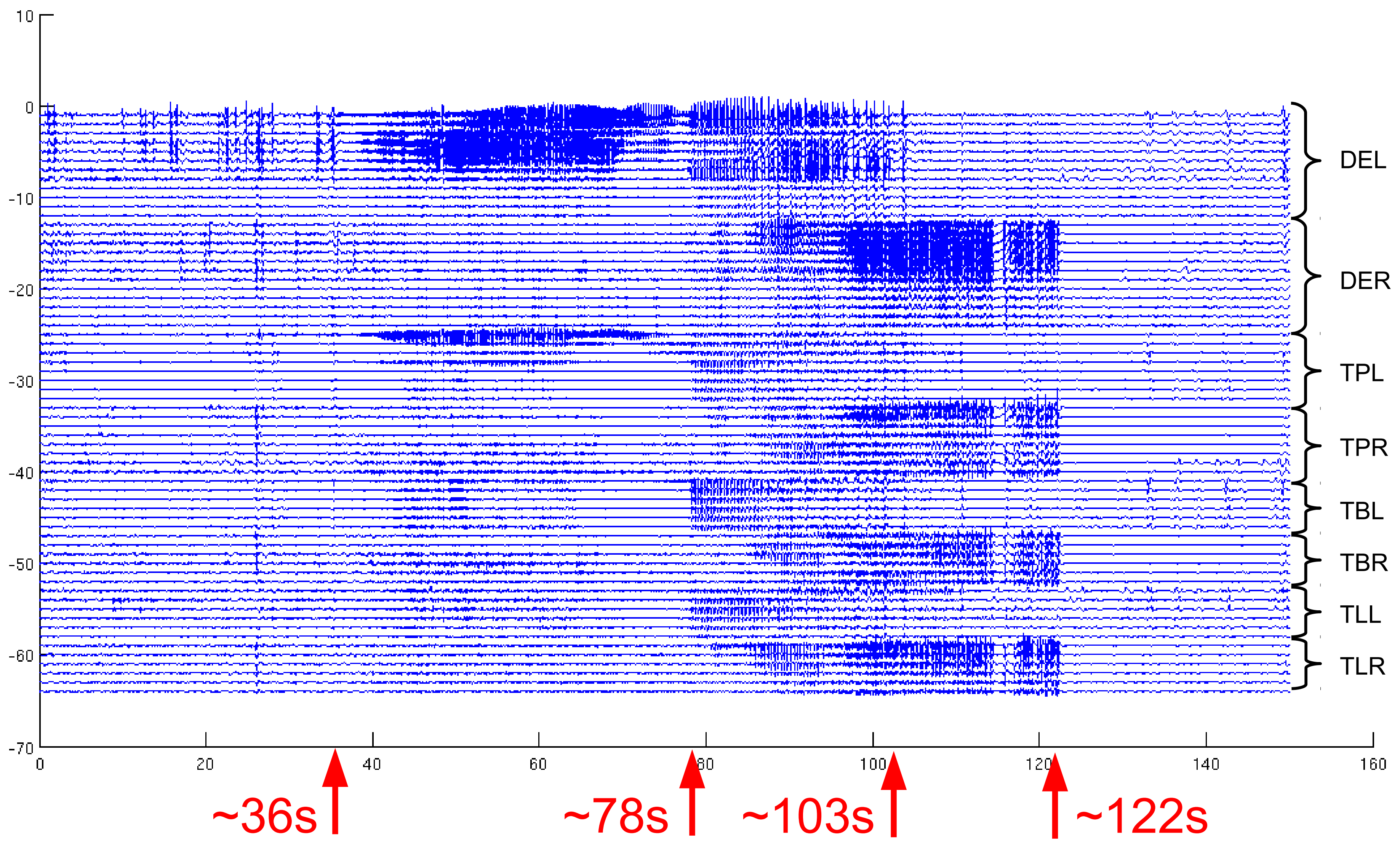


Example EEG

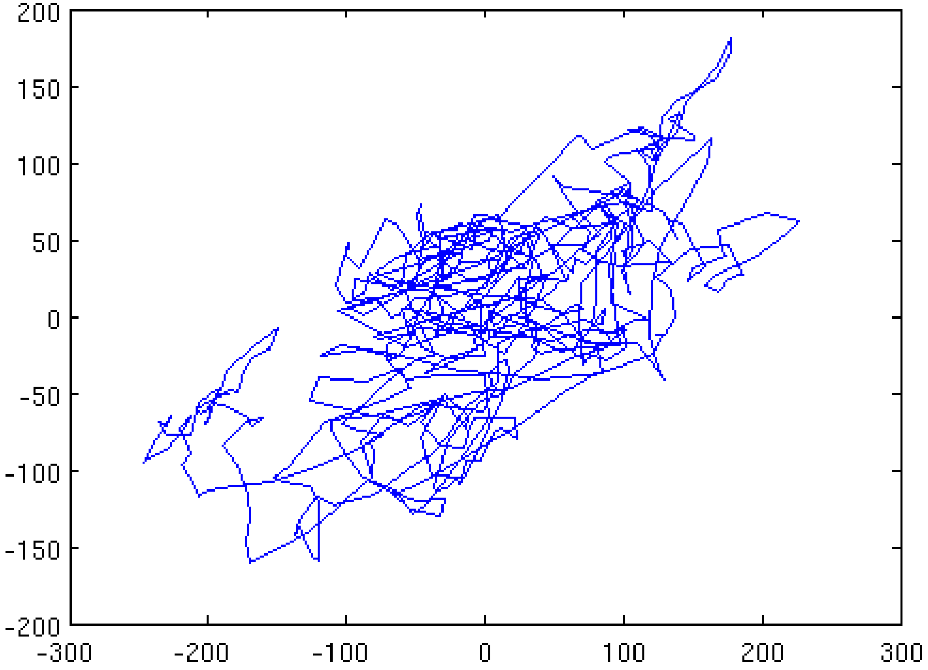



```
[tme,EEG] = display_EEG
('./data/','EEG_lecture.mat',128);
```

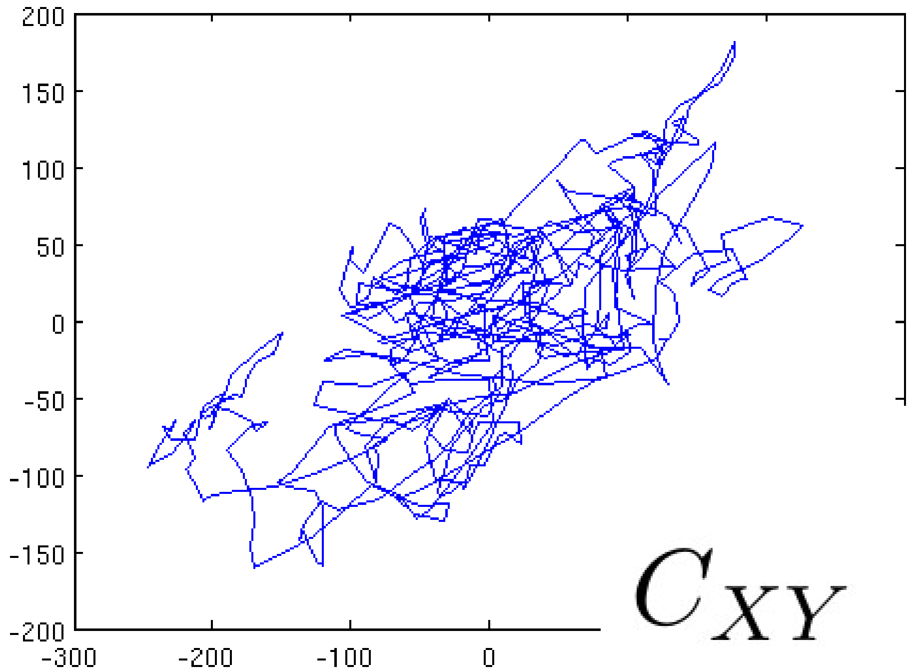
Example EEG


10
0
-10
-20
-30
-40
-50
-60
-70
0
20
40
60
80
100
120
140
160
DEL
DER
TPL
TPR
TBL
TBR
TLL
TLR
~36s
~78s
~103s
~122s

Covariance and Correlation



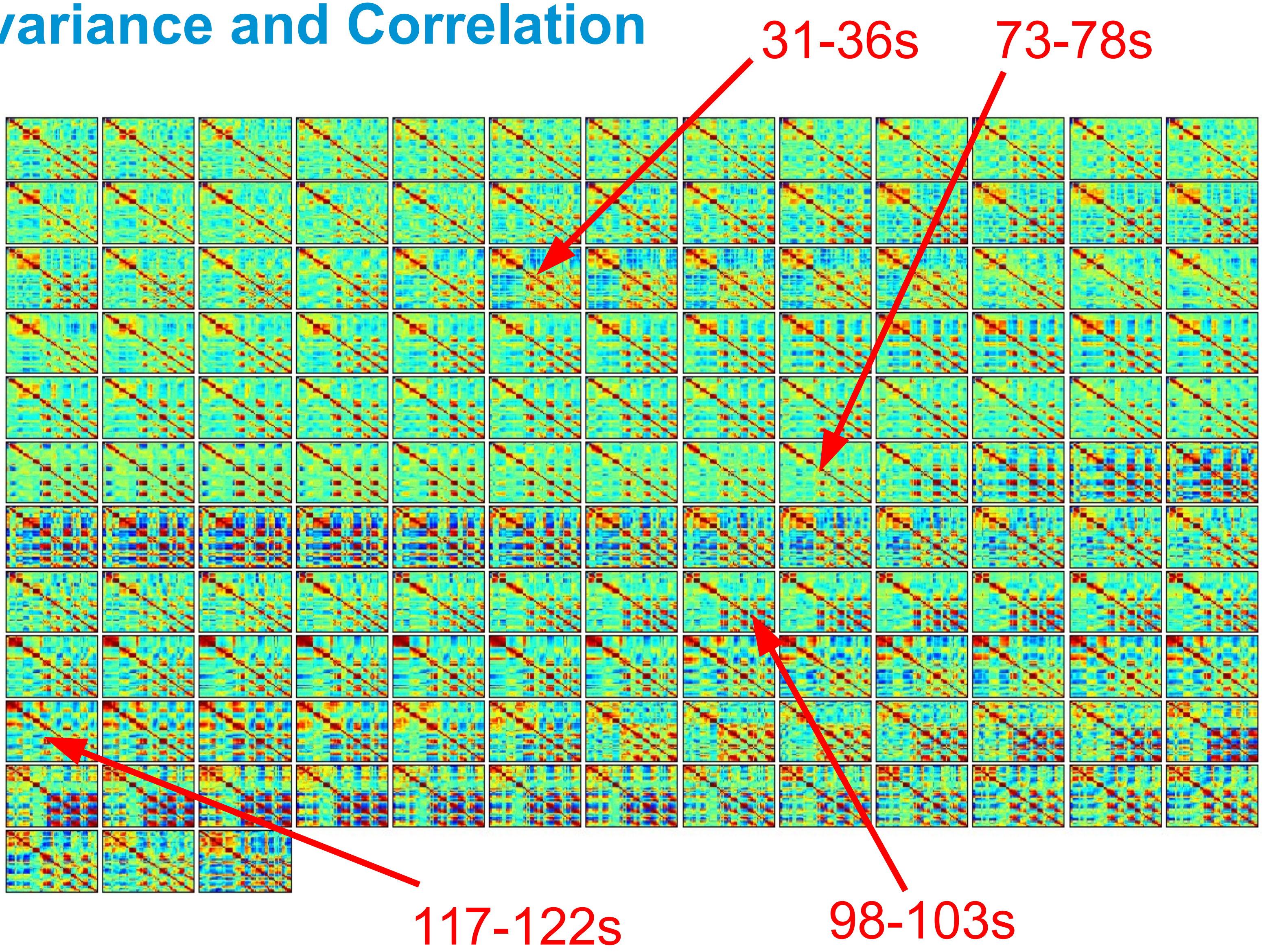
Covariance and Correlation



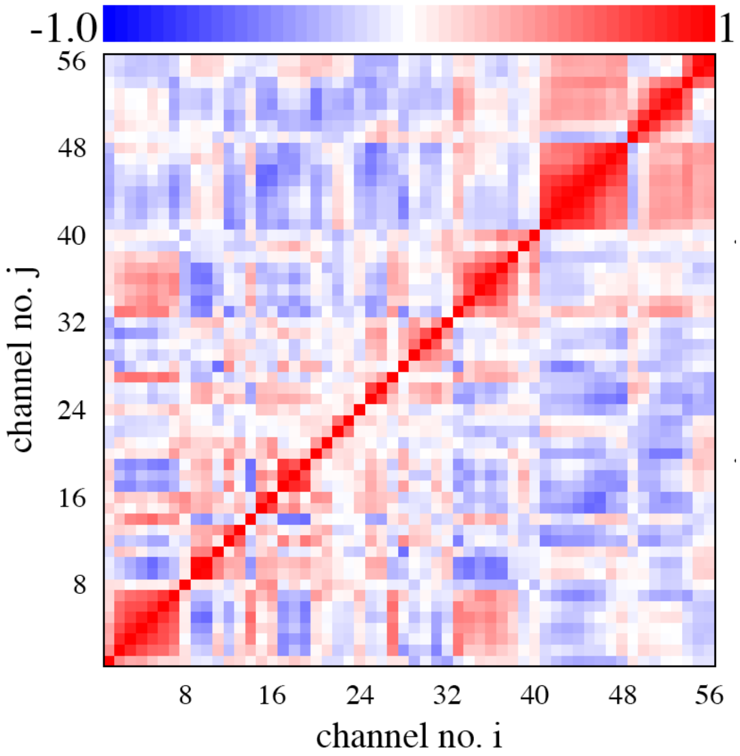
$$C_{XY} = \frac{1}{T}\sum_{t=1}^{T} \widetilde{X}_t \widetilde{Y}_t$$

```
CovVsCorr (tme,EEG, 640,128);
```

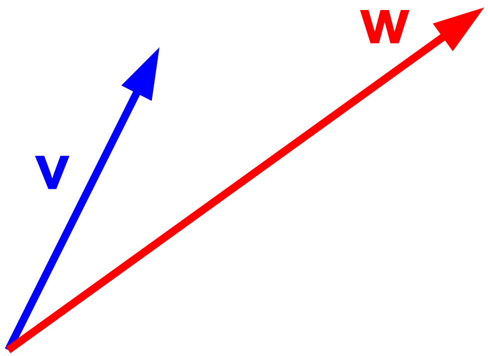
Covariance and Correlation



Eigenvalues and Eigenvectors



$$\mathbf{C}\,\mathbf{v} = \quad \mathbf{w}$$



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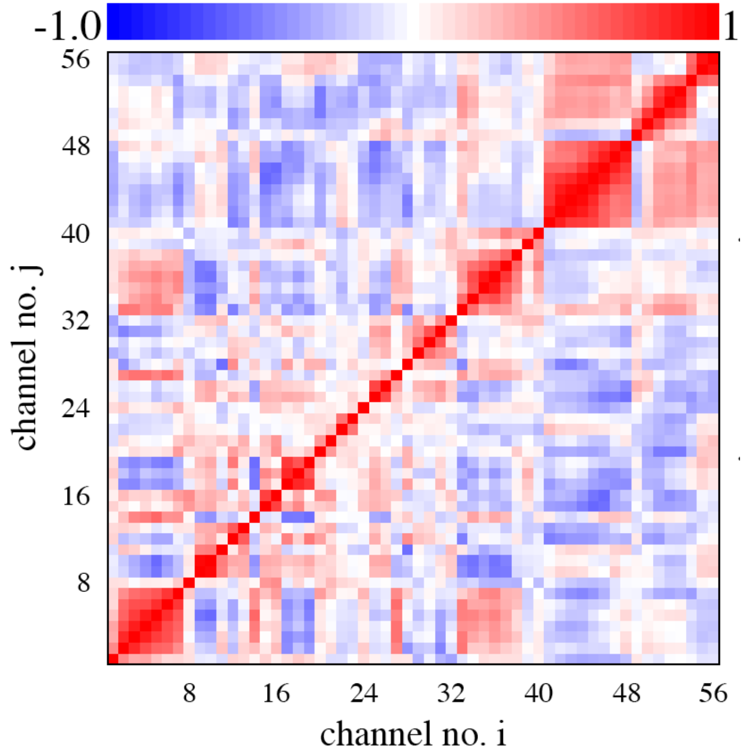
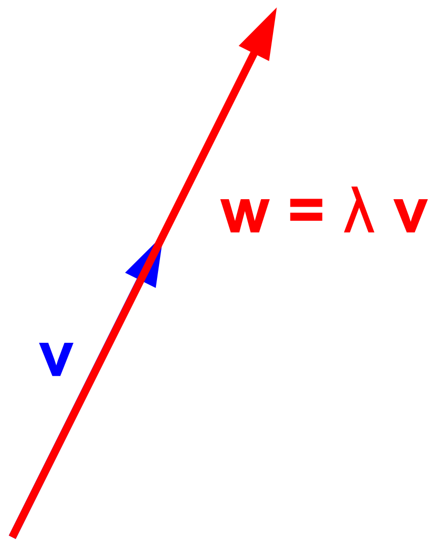


$$\mathbf{C}\,\mathbf{v} = \mathbf{w}$$

$$\mathbf{C}\,\mathbf{v} = \lambda\,\mathbf{v}$$



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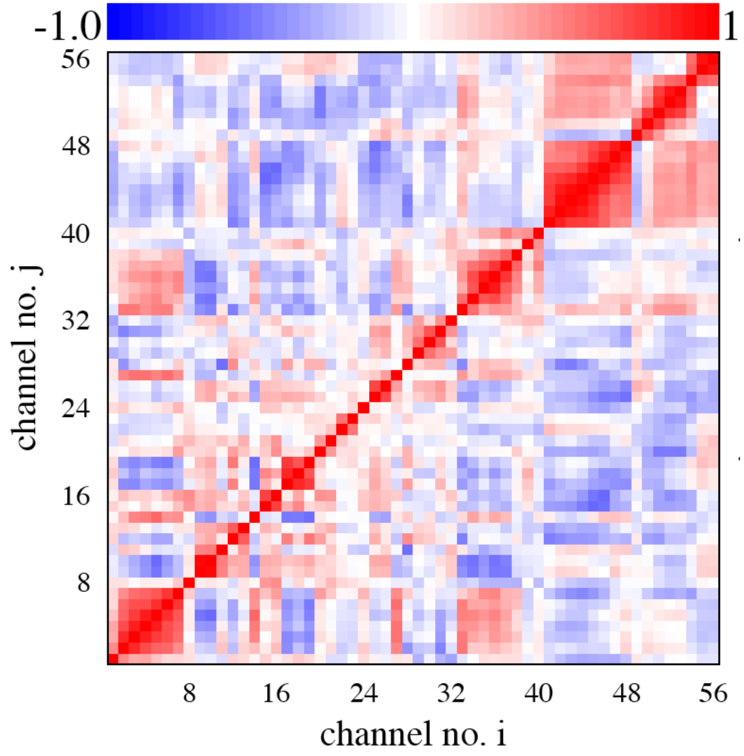

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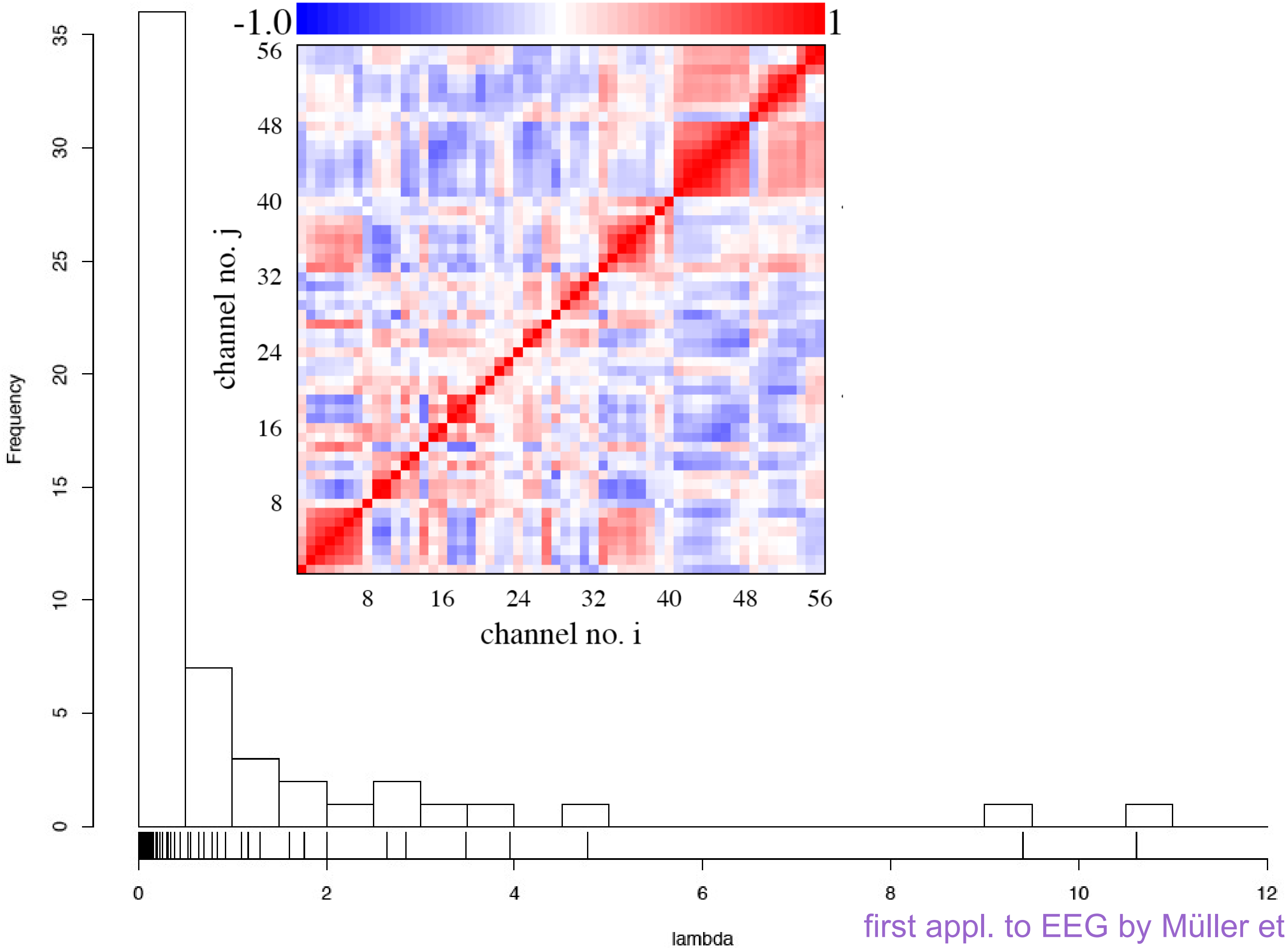
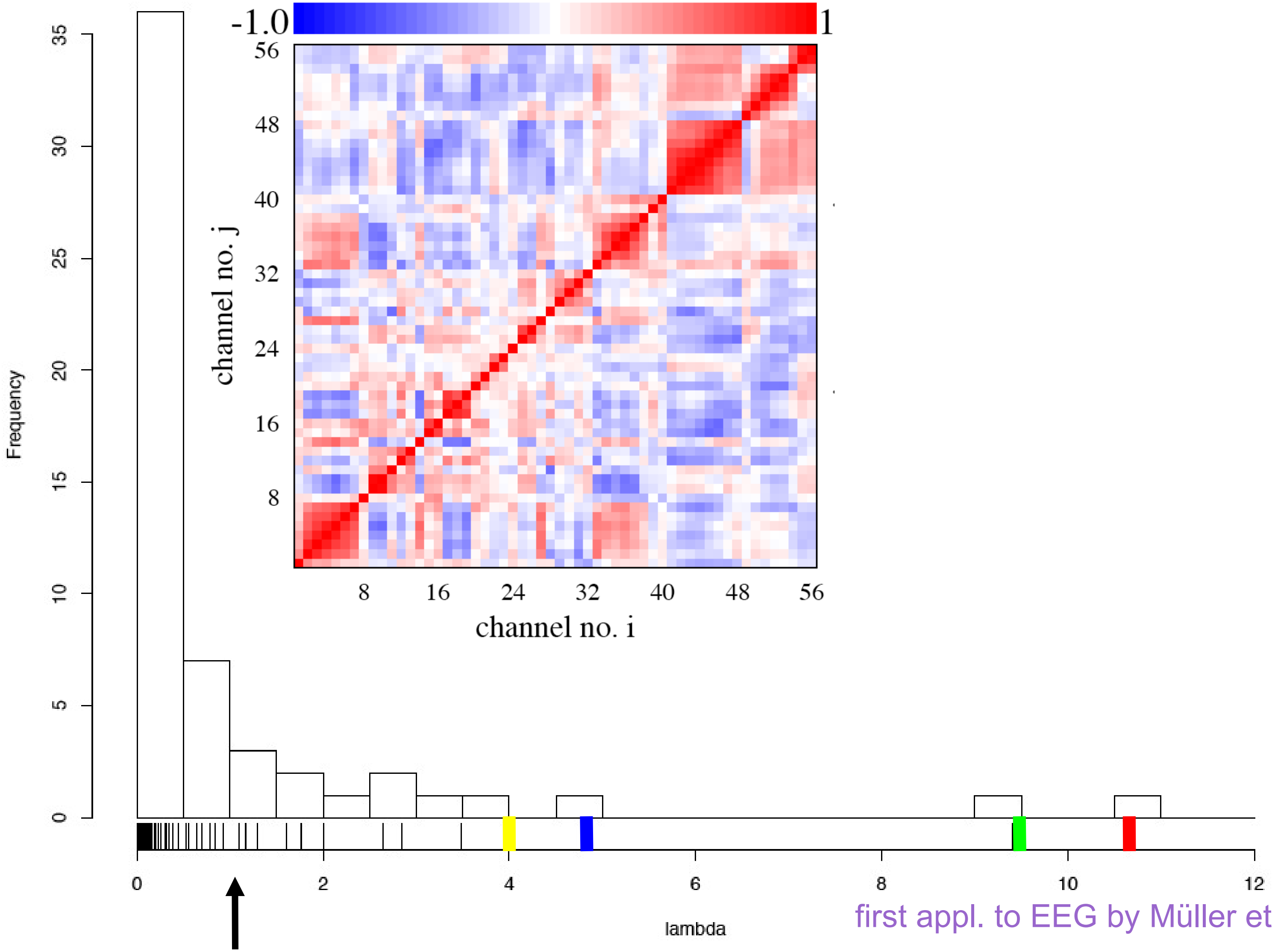


first appl. to EEG by Müller et al., Phys. Rev. E 71 (2005)

Eigenvalues and Eigenvectors



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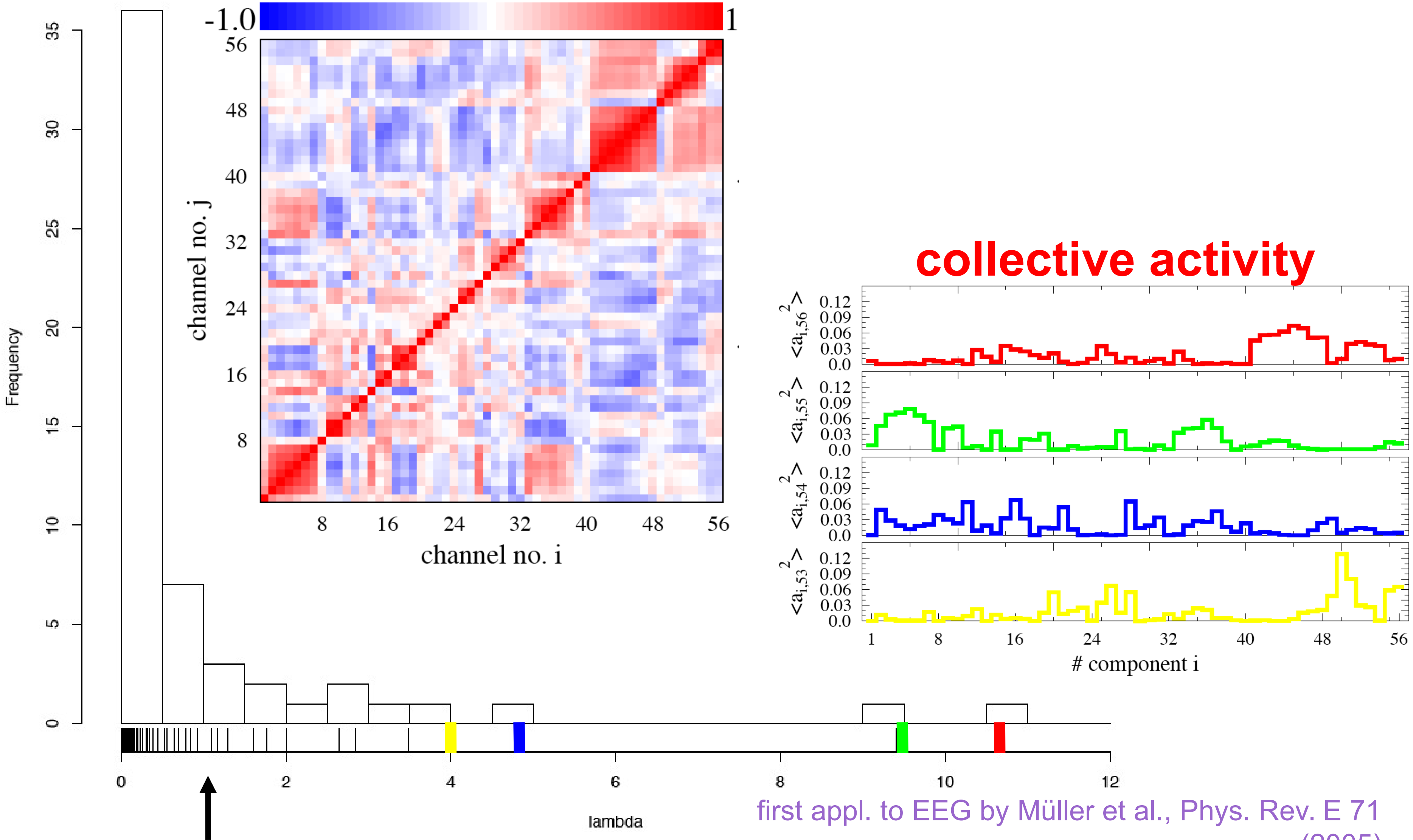
Eigenvalues and Eigenvectors



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Eigenvalues and Eigenvectors

properties of eigenvalues:

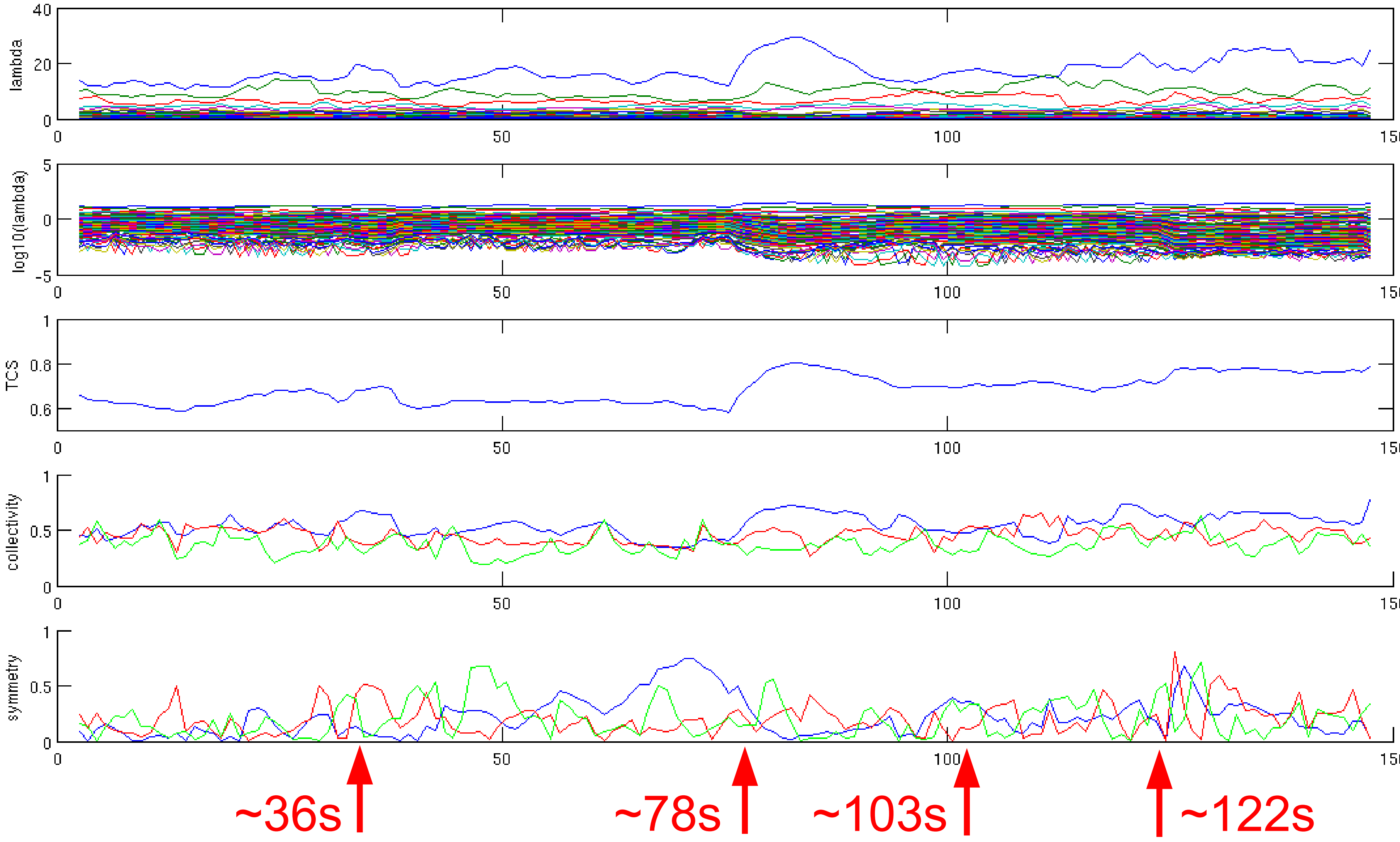
- can be sorted
- measure for total correlation strength (TCS)

properties of eigenvectors:

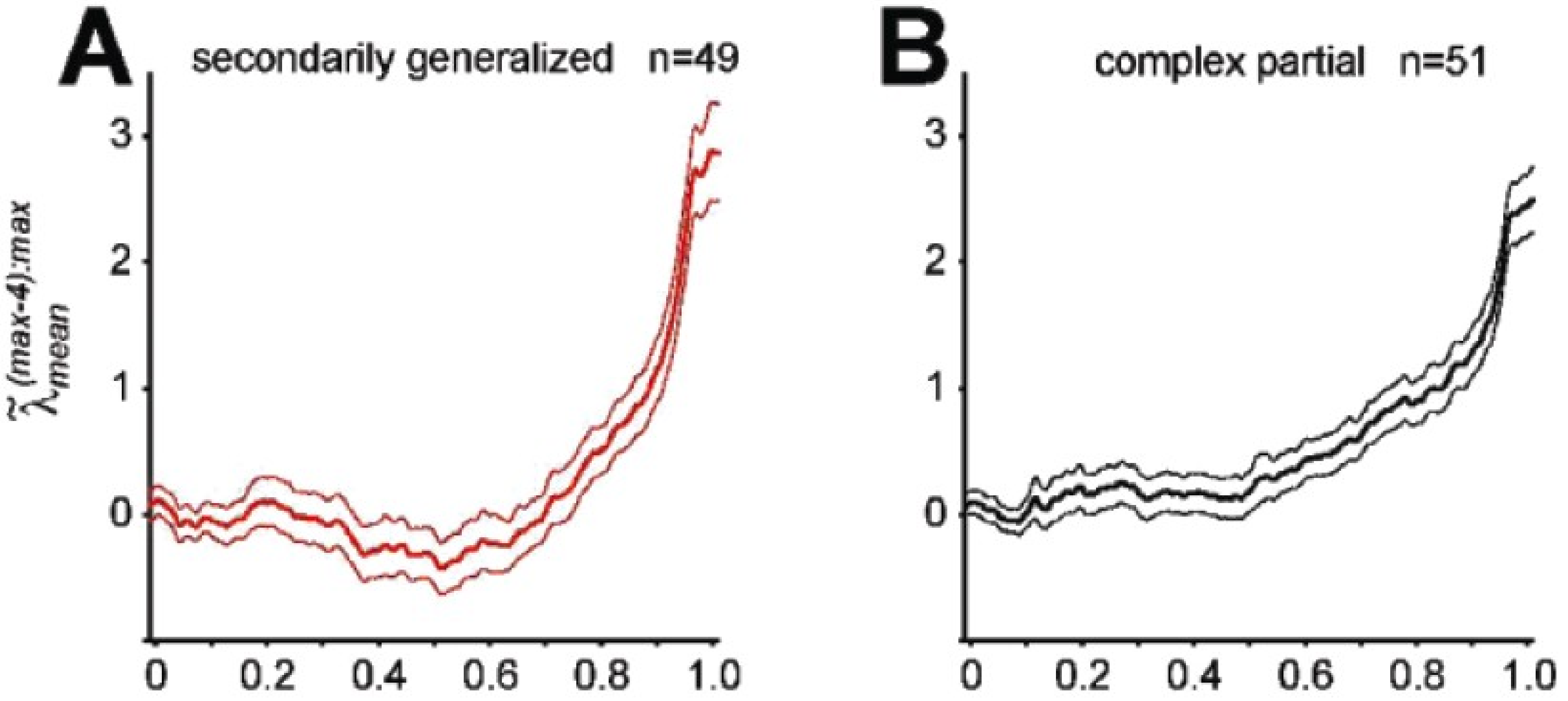
- collectivity
- symmetry

```
eig_val_vec (tme,EEG, 640,128);
```

Eigenvalues and Eigenvectors



Eigenvalues: correlation dynamics of seizures



Schindler et al., Brain 130 (2007)

Artificially correlated EEG

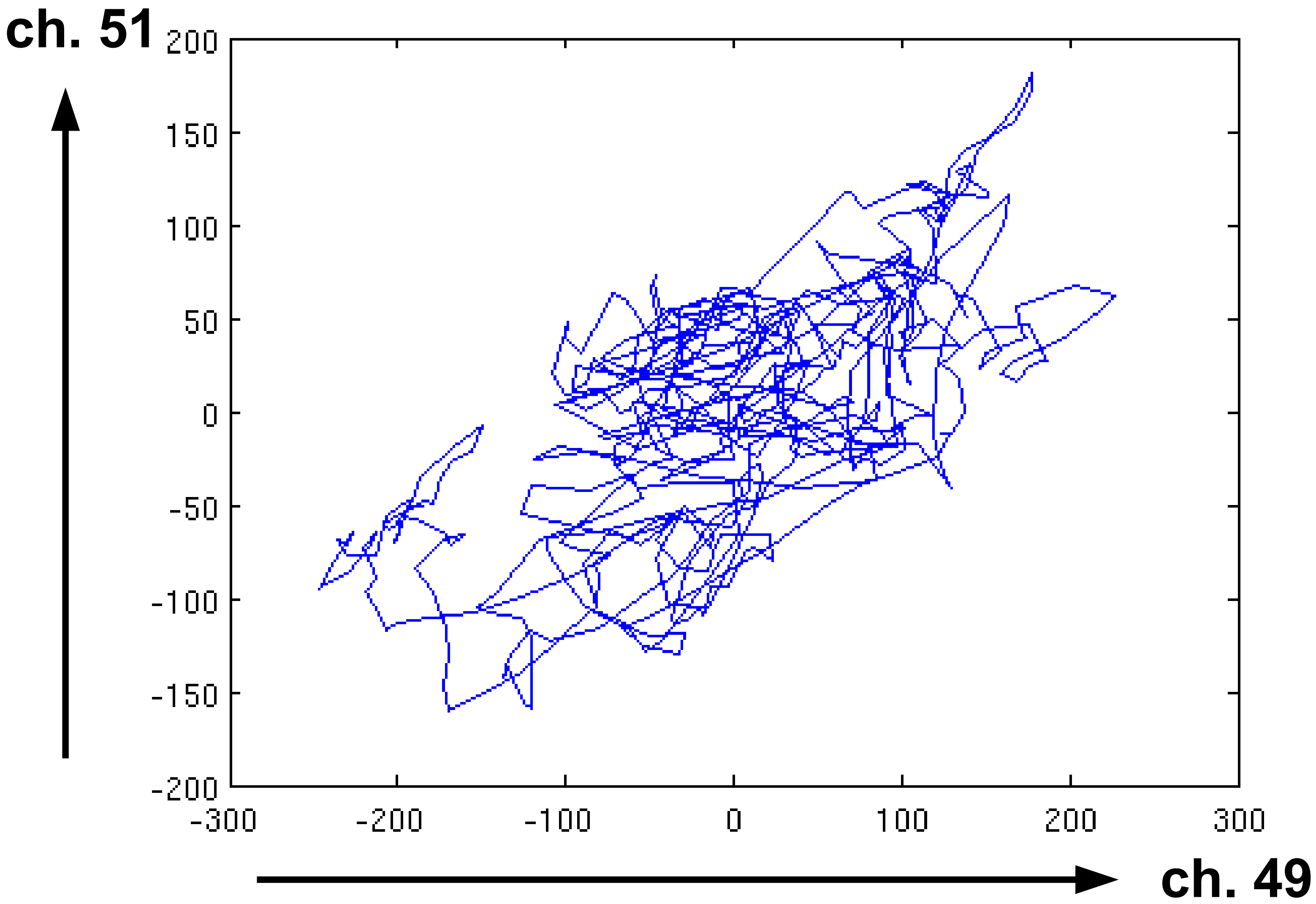
"black box" script

homework:

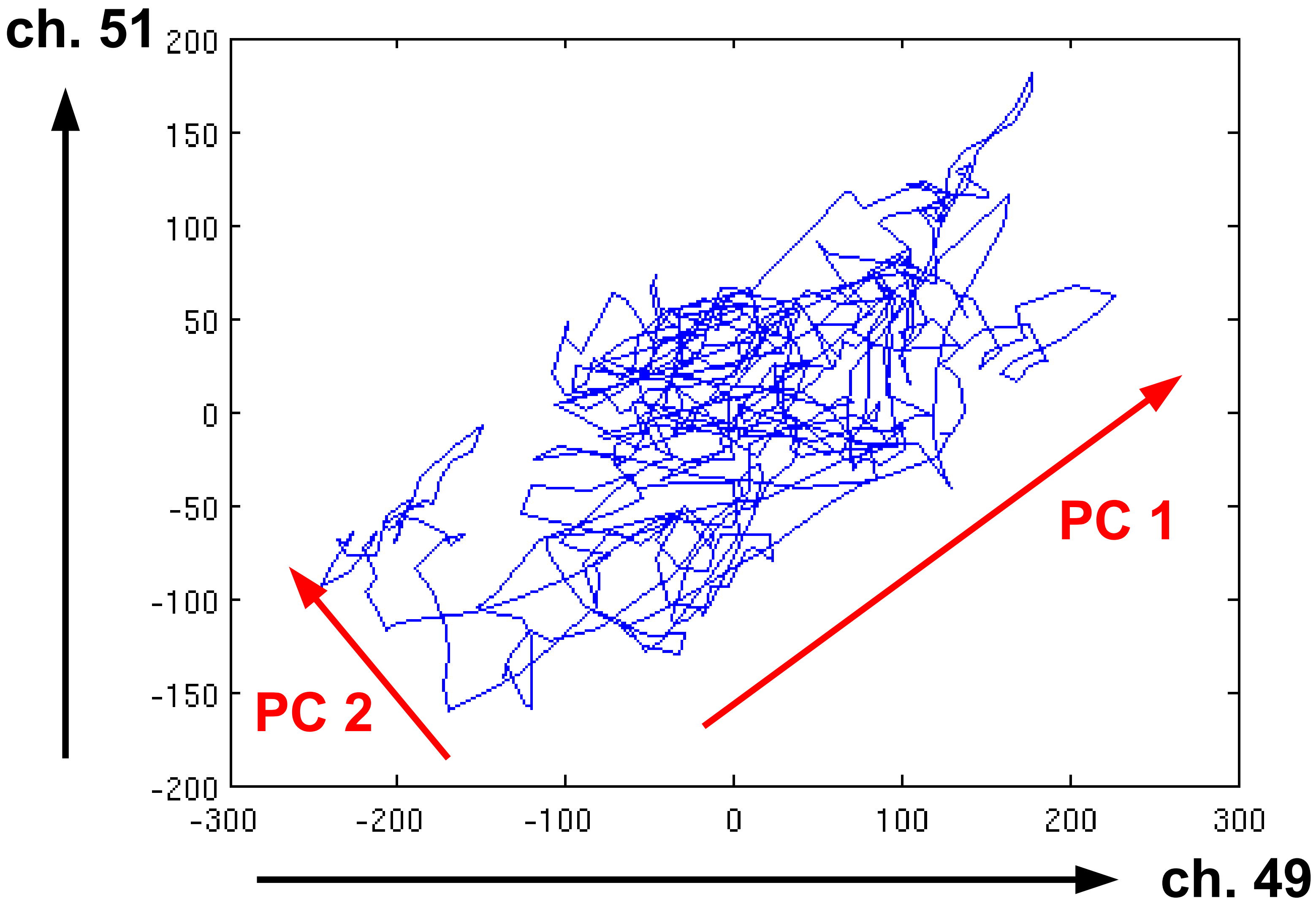
- analyze the script
- understand why it produces "EEG like" signals with desired arbitrary correlation pattern

```
EEG_blockcorr = blockcorr_EEG (EEG, corrpatt,SNR);
```

Principal Component Analysis (PCA)



Principal Component Analysis (PCA)



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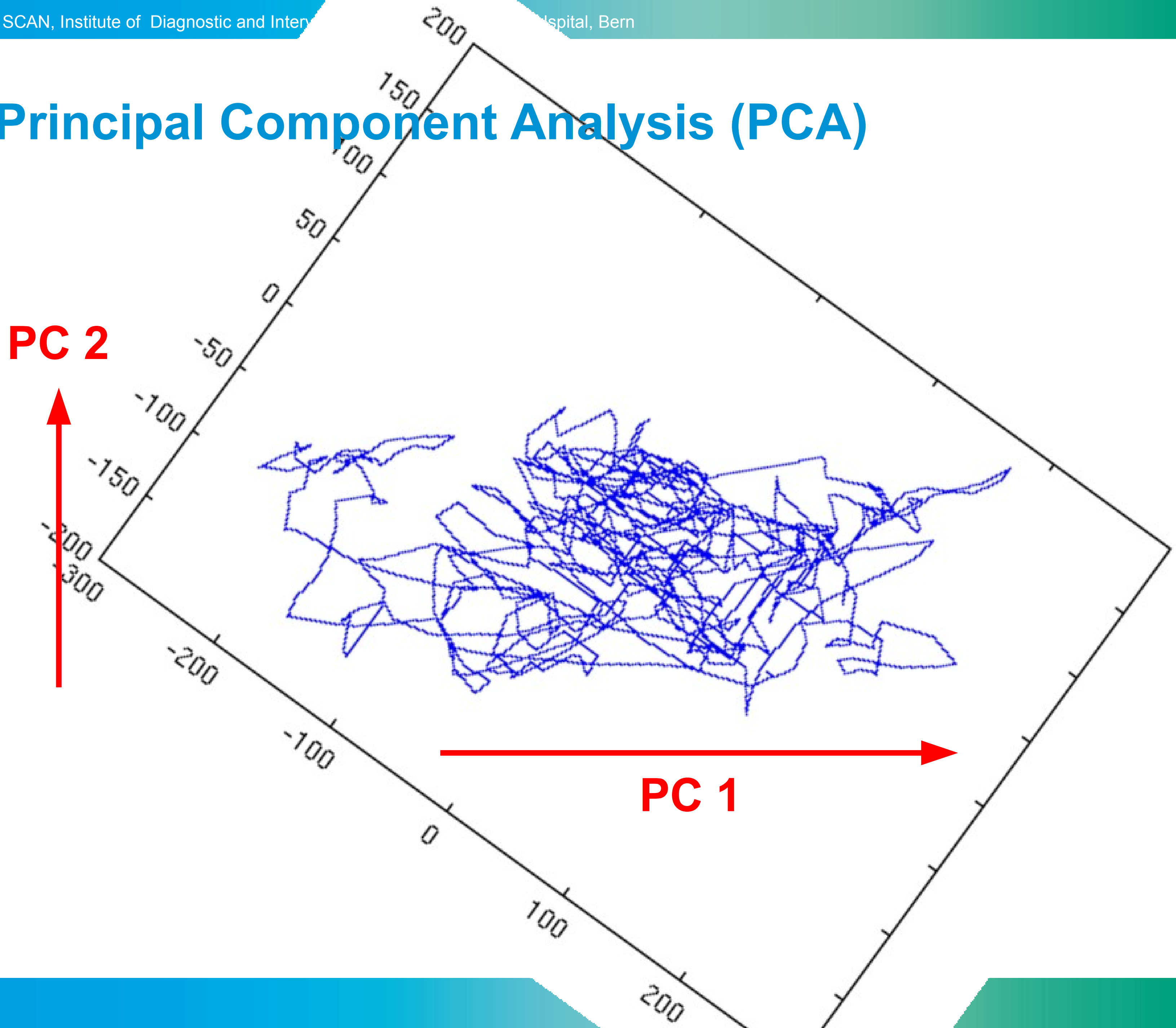

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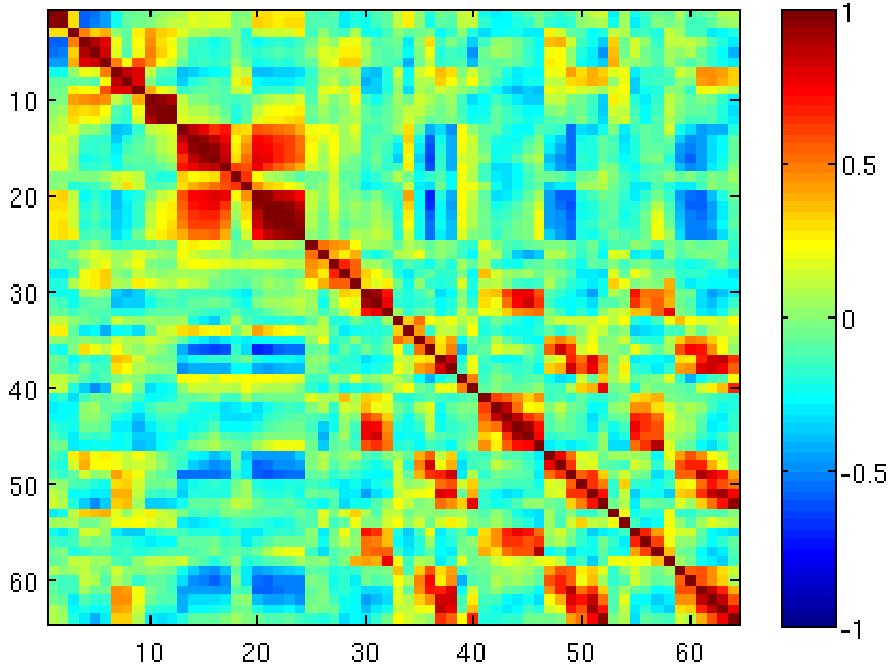

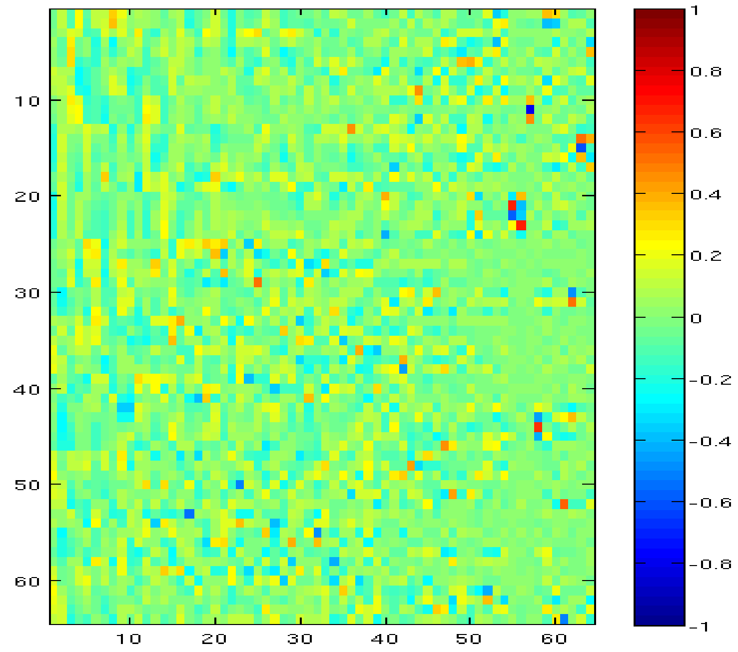


```
PrinCompAna (tme,EEG_blockcorr, PCA_del);
```

Literature suggestions

- Müller et al. (2005), Phys. Rev. E71, 046116.
- Müller et al. (2008), Europhys. Lett. 84, 10009.
- Rummel et al. (2013), Neuroinformatics 11, 159-173.
- Shlens (2014). A Tutorial on Principal Component Analysis.
- Schindler et al. (2007), Brain 131, 65-77.
- Schindler et al. (2007), Clin. Neurophysiol. 118, 1955-1968.

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Homework suggestions I

1) For the mathematically skilled ones:

   Go through slides 2 and 3. **Find the error**.

2) **Analyze** the uncommented Matlab function

   `blockcorr_EEG.m`:

   Which **mechanism** makes it possible to combine features of real EEG with a desired, arbitrary correlation pattern?

   Construct an artificial EEG with 19 channels and three uncorrelated blocks of size 7, 5 and 3.

   Analyze your artificial EEG with **`CovVsCorr.m`**.

   Check the dependence on the SNR.

Homework suggestions II

3) Use the Matlab functions **`CovVsCorr.m`** and **`eig_val_vec.m`** to **analyze** the data set **`EEG_homework.mat`**:
When and where do you think the **seizure starts**?
When do you think the **seizure terminates**?
Repeat analysis for first temporal **derivative** **`diff(EEG,1)`**.
Which seizure of the Supplementary Material of Rummel et al. (2013) is it?

Master and PhD theses

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