Functional networks

Frédéric Zubler

Benesco
18.12.2015

Definition of a graph

A graph is an ordered pair (V,E), where

- V is a set of elements called **vertices** or **nodes**

- E is a set of pairs (a,b) - with $a, b \in V$ - called **edges** or **links** or **connections.**

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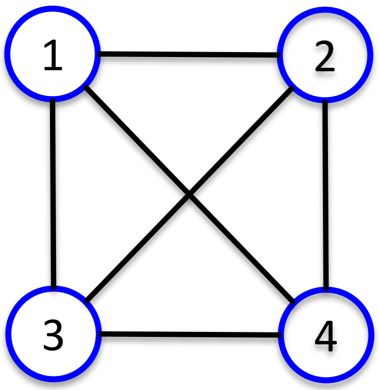
Example

G = (V,E) with

- V = {1, 2, 3, 4}

- E = {(1,2), (1,3), (1,4), (2,3), (2,4), (3,4)}



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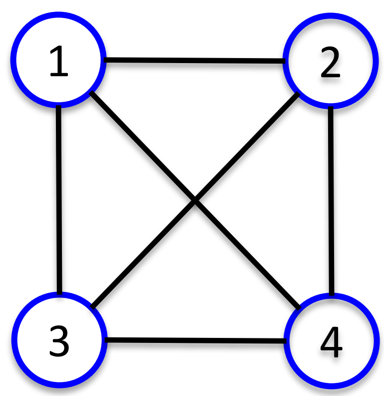
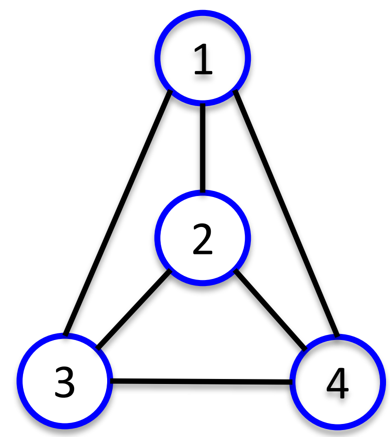
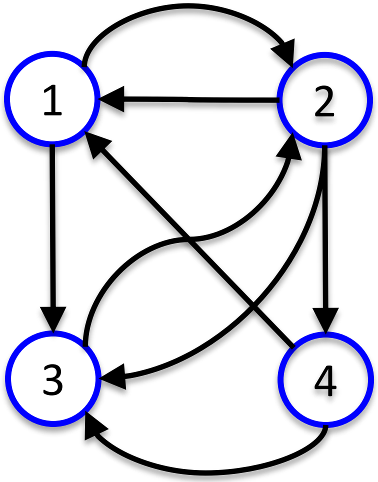




Some definitions

A graph is **directed** (or **oriented**) if E is a set of *ordered* pairs

i.e. (a,b) ≠ (b,a).



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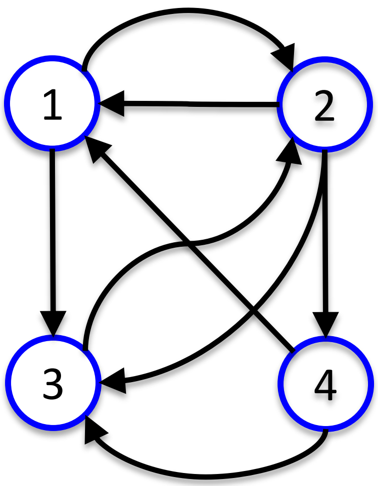
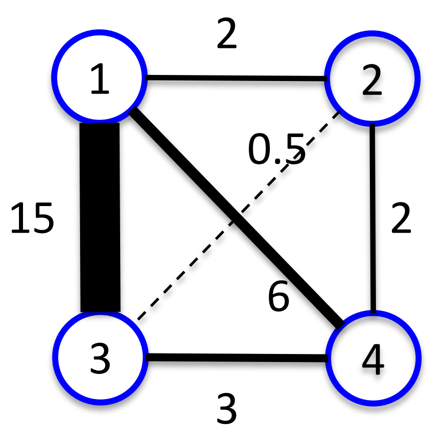
A graph is **weighted** if a label (weight) is associated with each edge.





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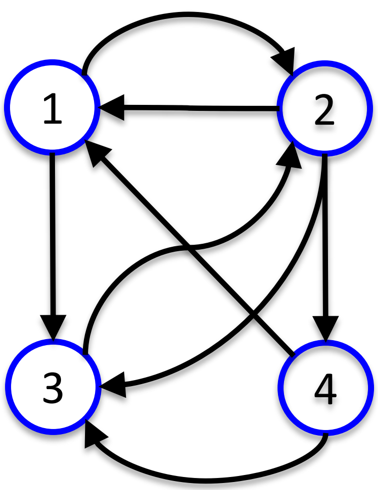
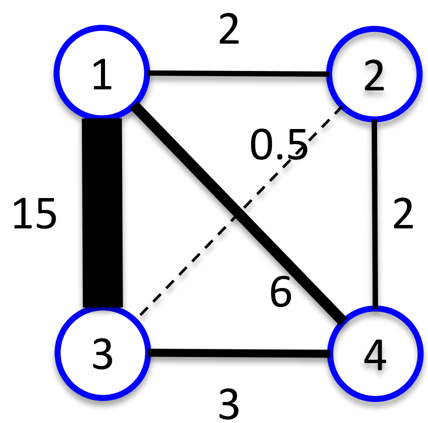
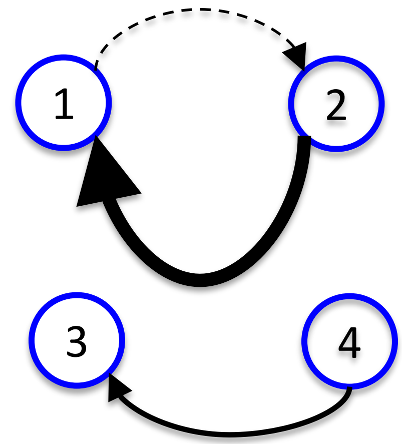
A graph is **connected** if there is a path from any node to any other node.



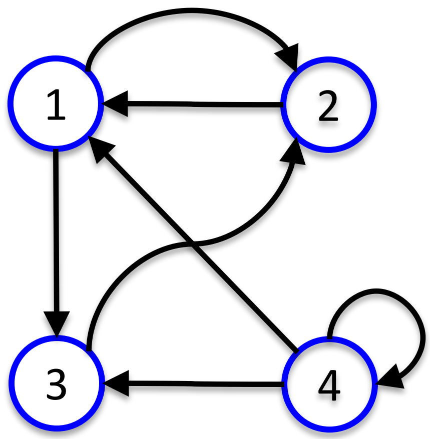




Representation with a matrix

The **adjacency matrix** of a graph G is a square matrix A such that

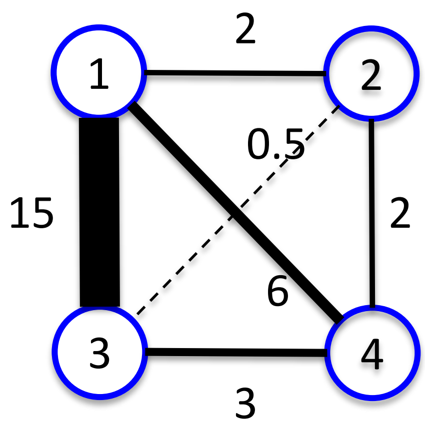
A[*i*,*j*] is the weight of the edge from vertex *i* to vertex *j*



to

from

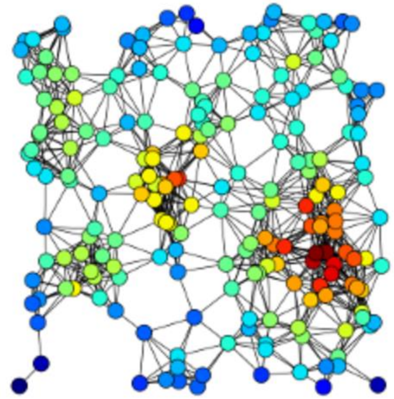
0	1	1	0
1	0	0	0
0	1	0	0
1	0	1	1



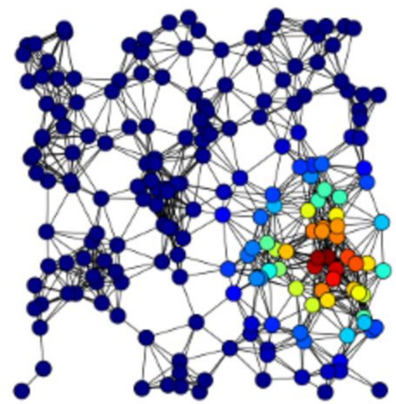
0	2	15	6
2	0	0.5	2
15	0.5	0	0
6	2	1	0

Identifying important nodes

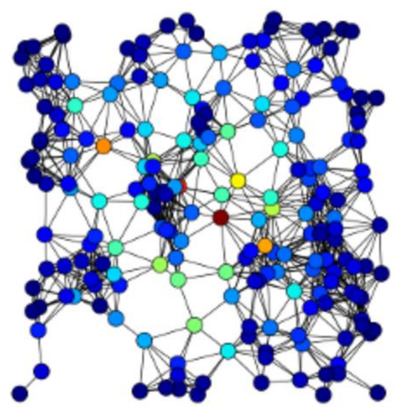
Degree centrality



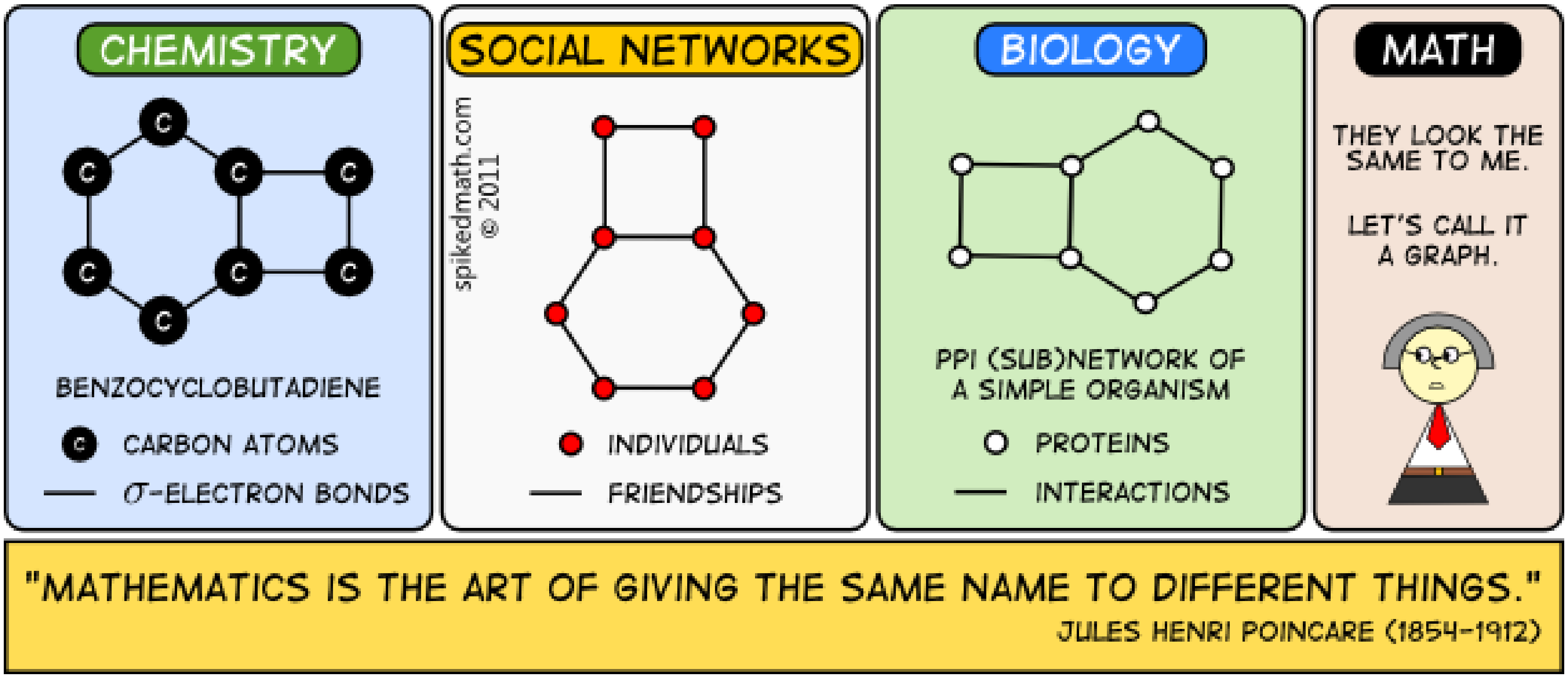
Eigenvector centrality



Betweenness centrality



wikipedia

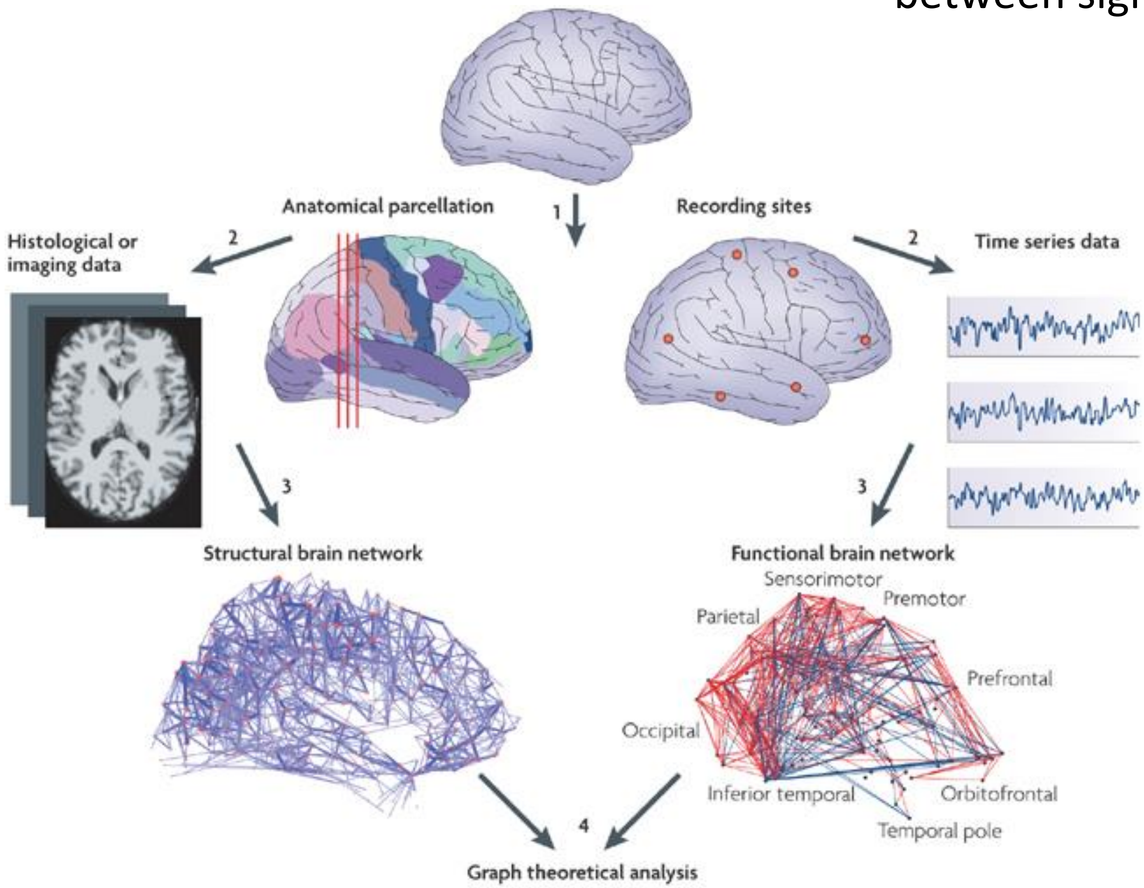


spikedmath.com

2 types of graphs in Neuroscience

structural networks :
nodes = brain areas, neurons, ...
links = anatomical connections

functional networks :
nodes = signals (EEG, fMRI,...)
links = mathematical relation between signals



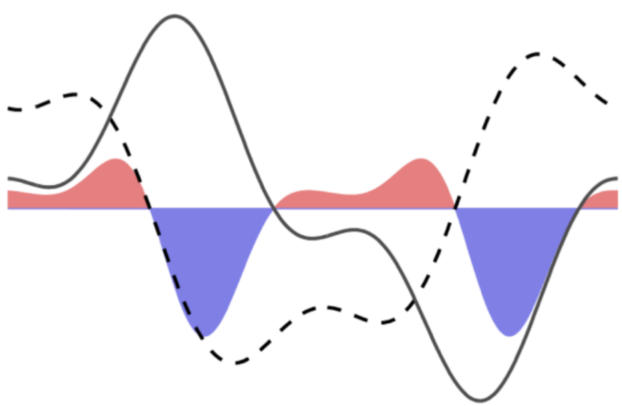
Bullmore & Sporns, 2009

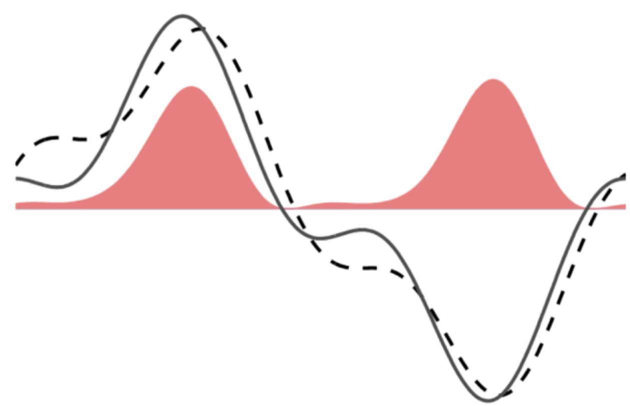
Cross-correlation function:

$$x = (x_1, x_2, \dots, x_N)$$

$$y = (y_1, y_2, \dots, y_N)$$

$$C(x,y) = \frac{1}{N}\sum_{i=1}^{N} \frac{(x_i - m_x)}{\sigma_x}\frac{(y_i - m_y)}{\sigma_y}$$





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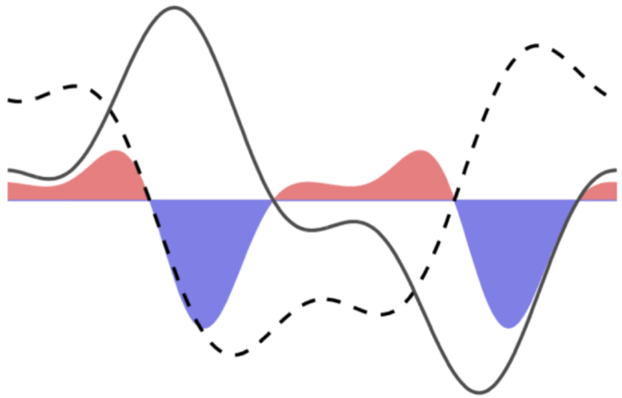
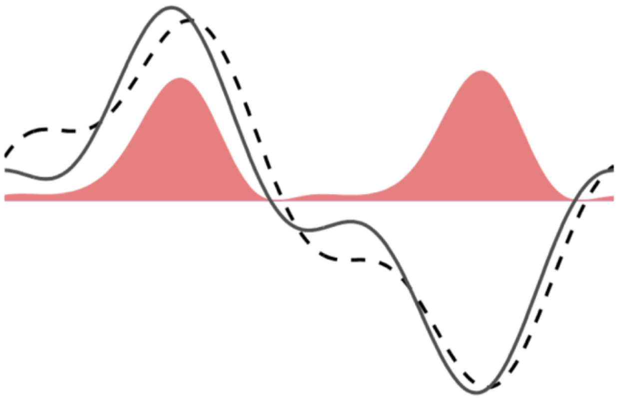
C = crosscoef(A);





First test in Matlab

```
% 5 signals
s1 = rand(1000,1);
s2 = 0.1 * s1;
s3 = s1 + rand(1000,1);
s4 = rand(1000,1);
s5 = s1 + 0.4*s4;

% compute correlation
C = corrcoef([s1,s2,s3,s4,s5]);
C = abs(C);

% plot
imagesc(C);
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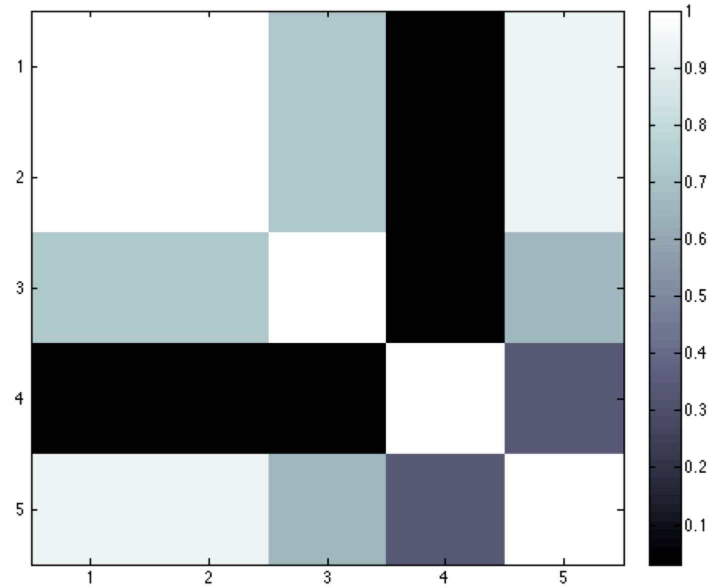
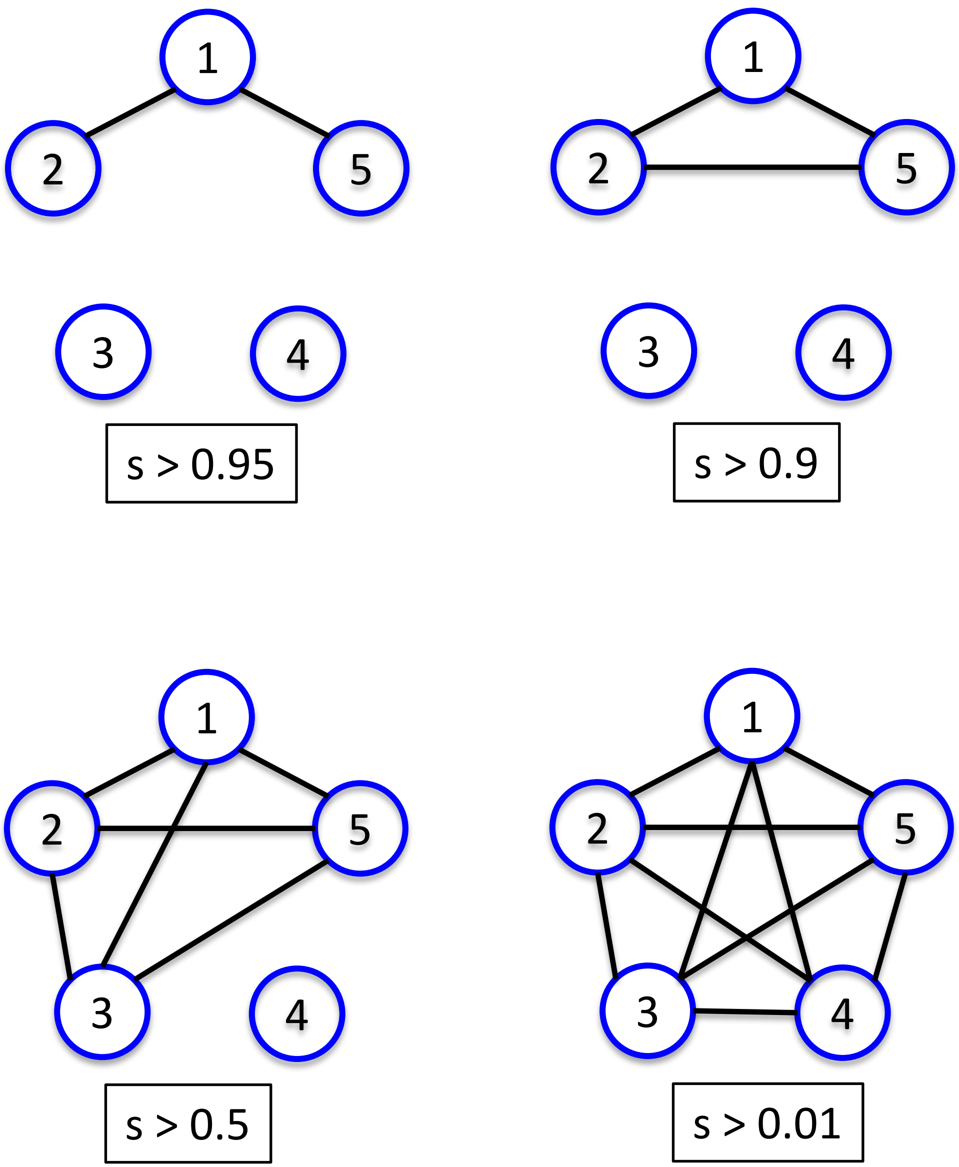
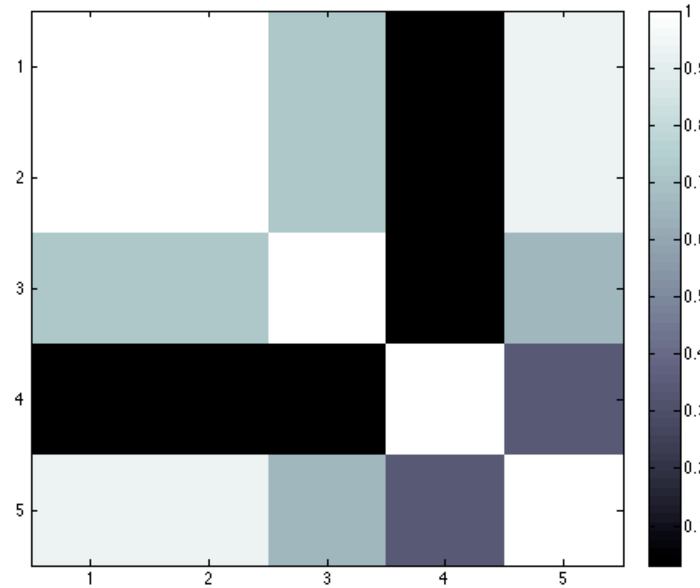


Binary graphs: thresholding





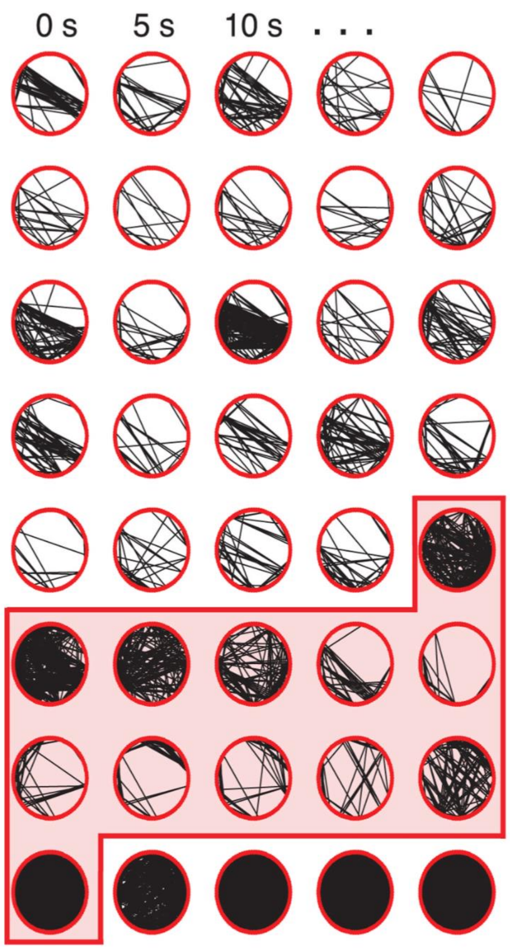
1	1	0.71	0.03	0.93
1	1	0.71	0.03	0.93
0.71	0.71	1	0.05	0.66
0.03	0.03	0.05	1	0.34
0.93	0.93	0.66	0.34	1

Coalescence and Fragmentation of Cortical Networks during Focal Seizures

Mark A. Kramer[1], Uri T. Eden[1], Eric D. Kolaczyk[1], Rodrigo Zepeda[2], Emad N. Eskandar[3,4], and Sydney S. Cash[2,4]

+ Show Affiliations

The Journal of Neuroscience, 28 July 2010, 30(30): 10076-10085; doi: 10.1523/JNEUROSCI.6309-09.2010



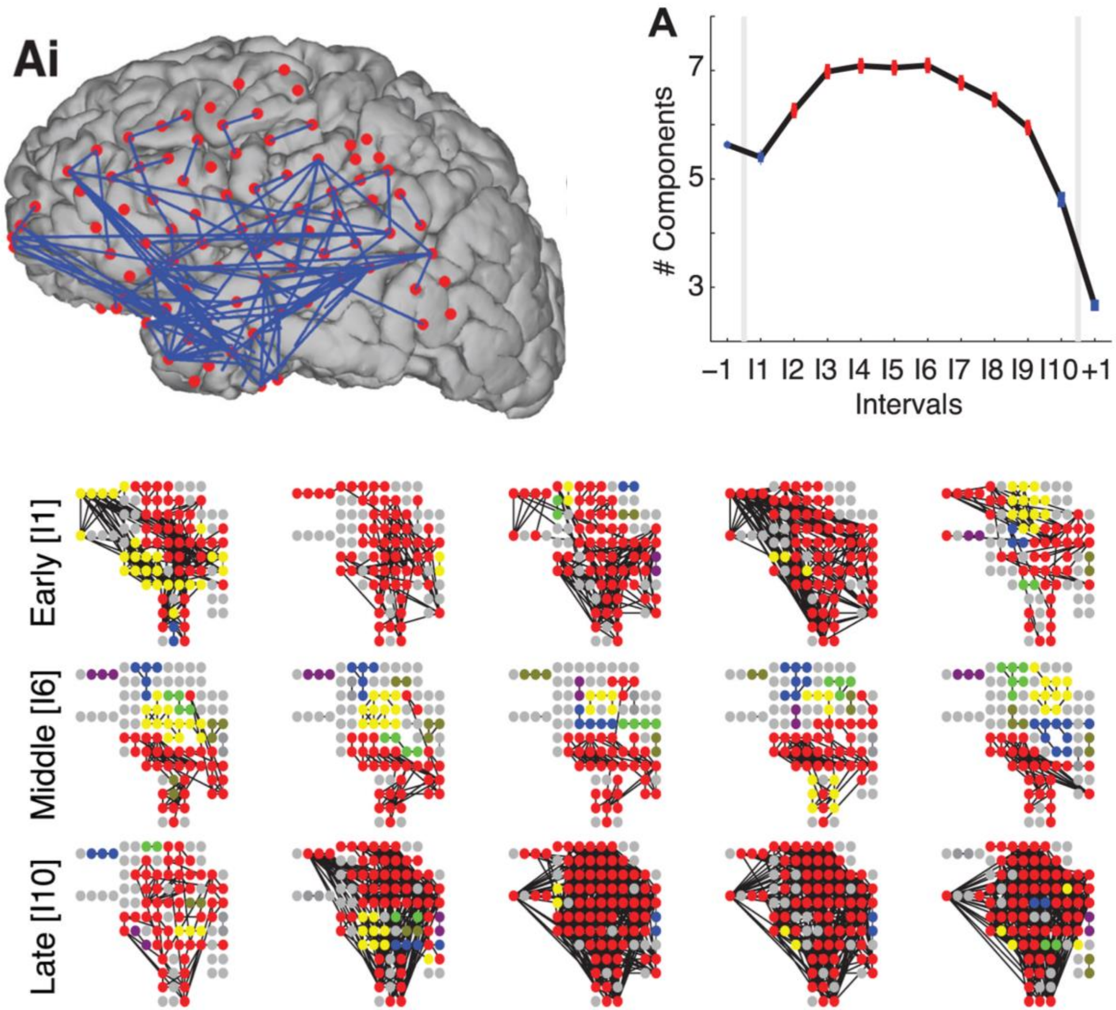


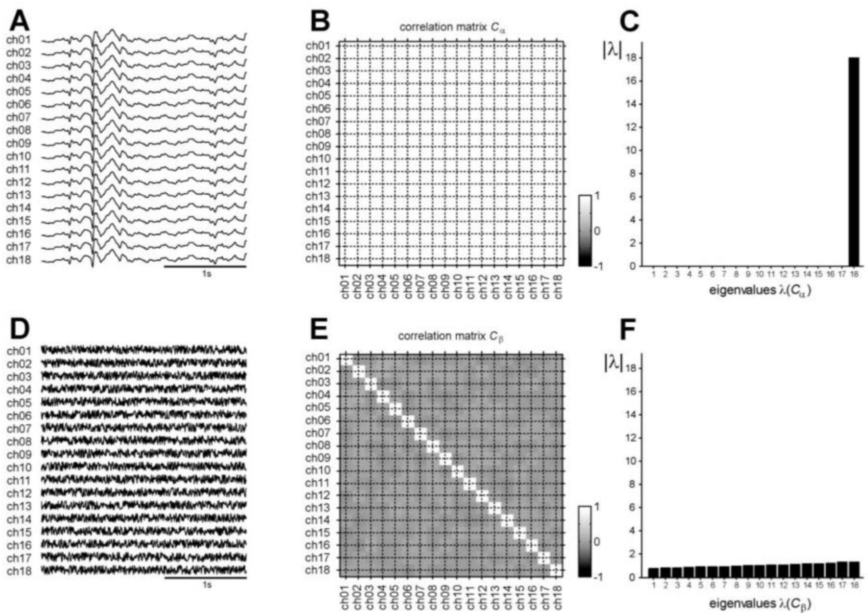
Figure 2

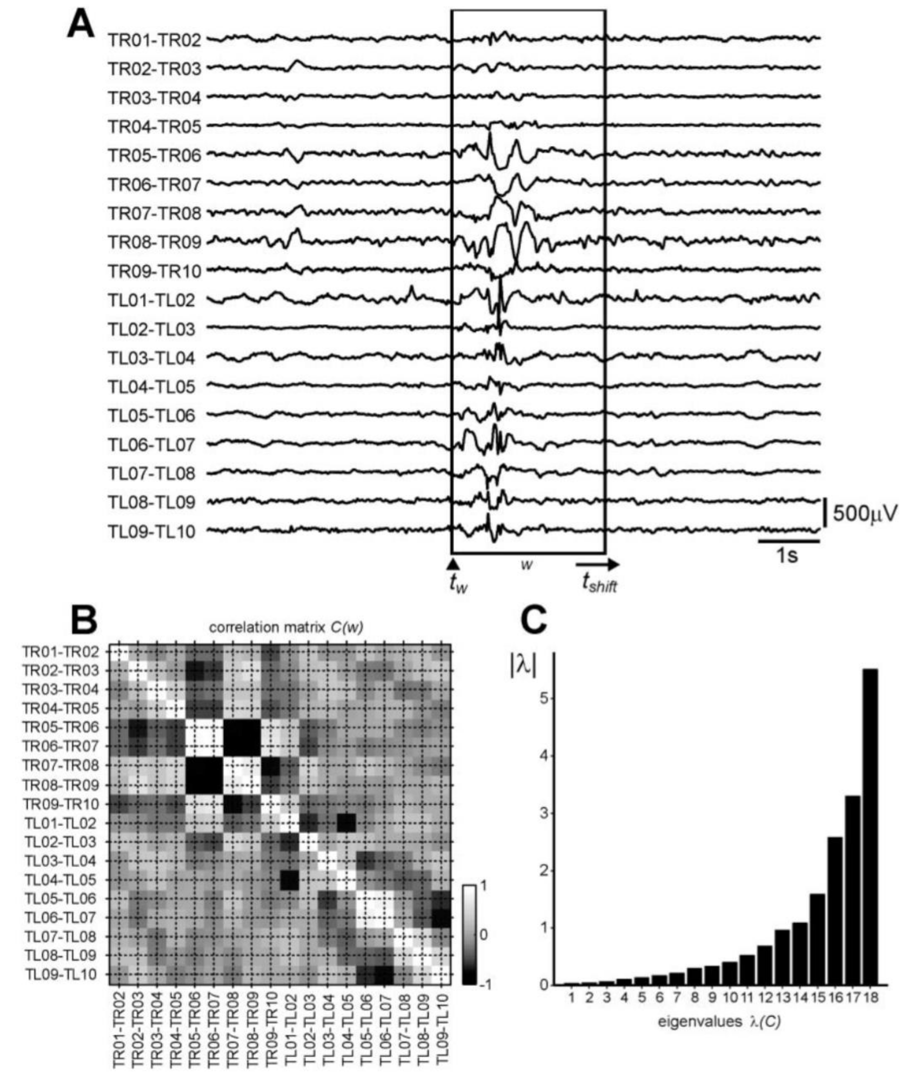
doi:10.1093/brain/awl304

Brain (2007), **130**, 65–77

Assessing seizure dynamics by analysing the correlation structure of multichannel intracranial EEG

Kaspar Schindler,[1] Howan Leung,[1] Christian E. Elger[1] and Klaus Lehnertz[1,2]





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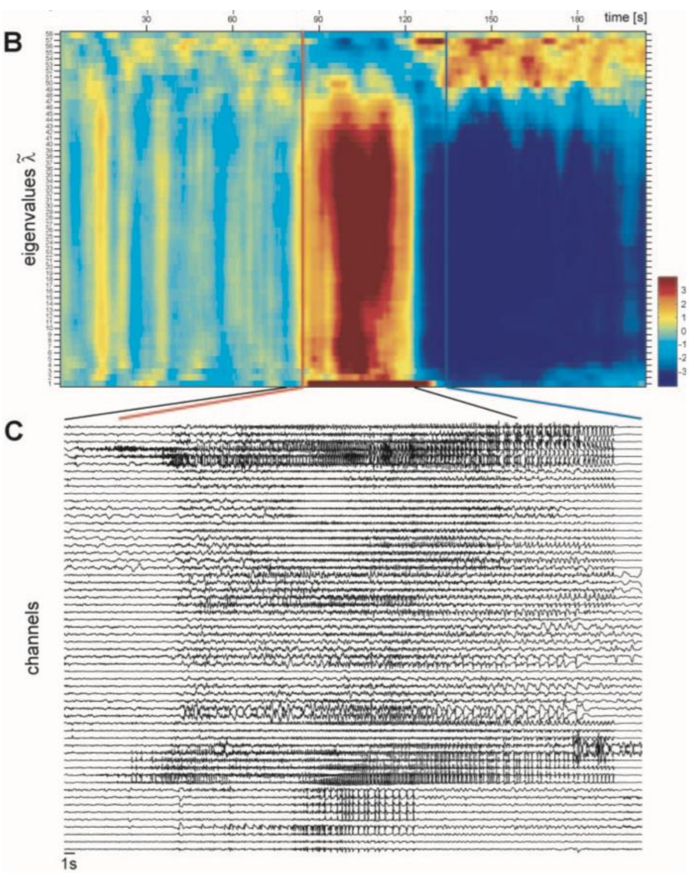
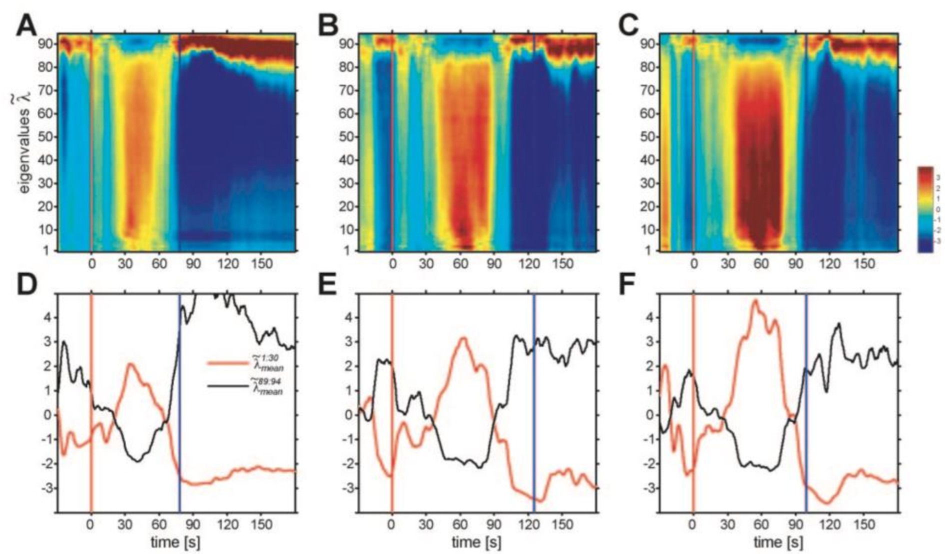




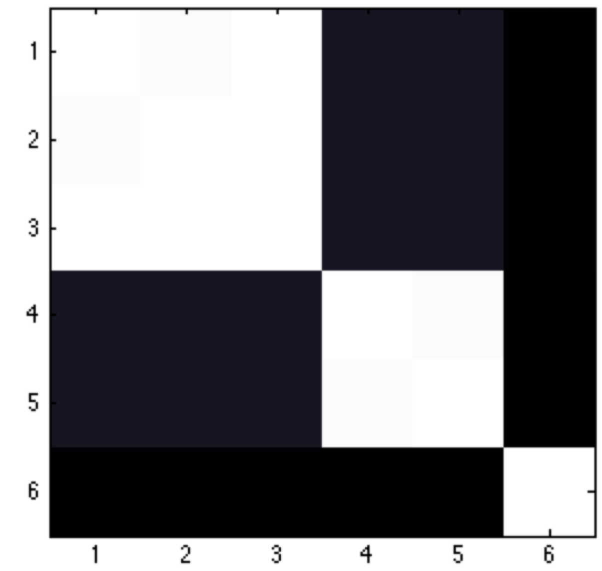
Eigenvectors of the correlation matrix

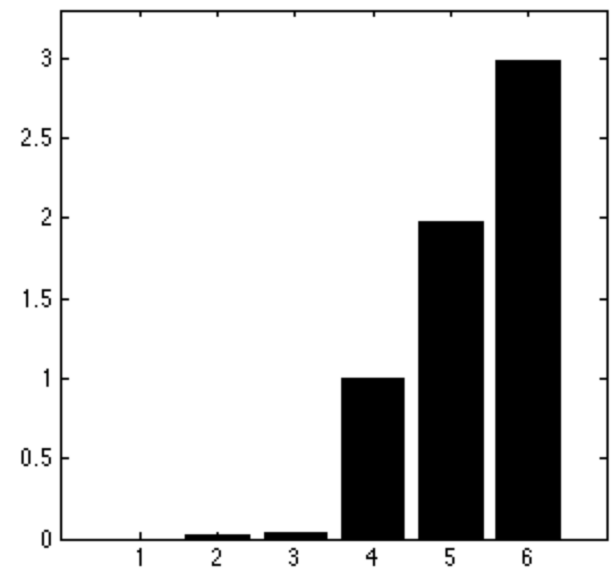
```
% 2 mini clusters and one isolated signal
s1 = rand(1000,1);
s2 = 5*s1 + rand(1000,1);
s3 = s1 + s2;
s4 = rand(1000,1);
s5 = 4*s4+rand(1000,1);
s6 = rand(1000,1);

% compute correlation matrix
C = corrcoef( [s1,s2,s3,s4,s5,s6] );
C = abs(C);

% compute and sort eigenvectors
lambda = eig(C);
lambda= abs(lambda);
lambda= sort(lambda);

% PLOT
subplot(2,1,1);
imagesc(C); colormap(bone); caxis([0,1])
subplot(2,1,2);
bar(lambda); xlim([0,7])
```

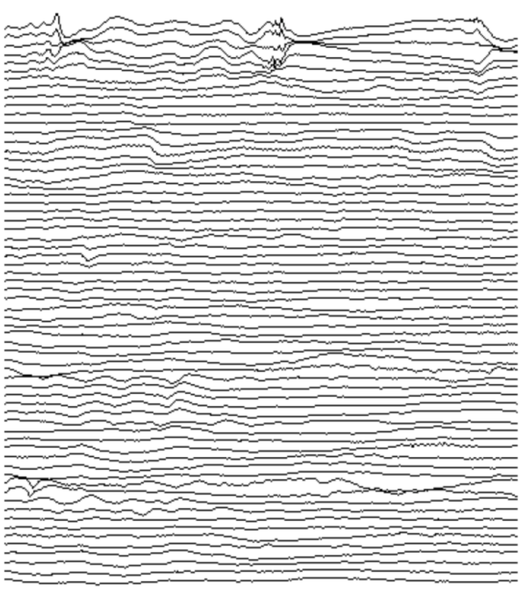




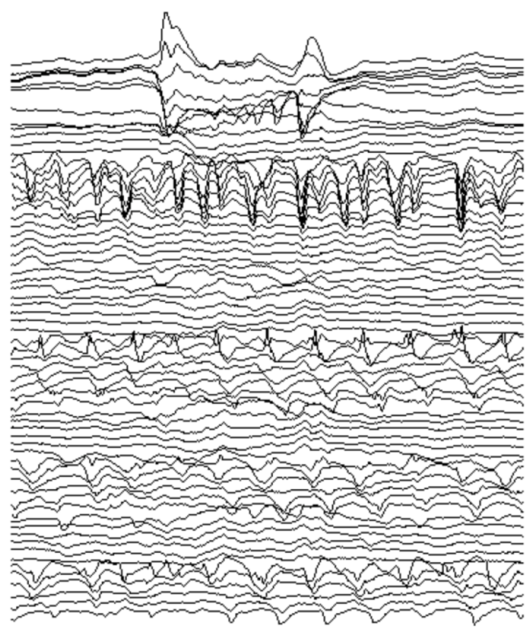
Exercise

iEEG data, 3 epochs of 2 secs at 500Hz (=1000 sampling points)
for each epoch:

a) compute correlation matrix.
b) find the channel with highest degree centrality.
c) find the 10 channels which are the most correlated with it.
d) compute the ratio 5 largest / 60 smallest eigenvalues.



eeg_pre.mat



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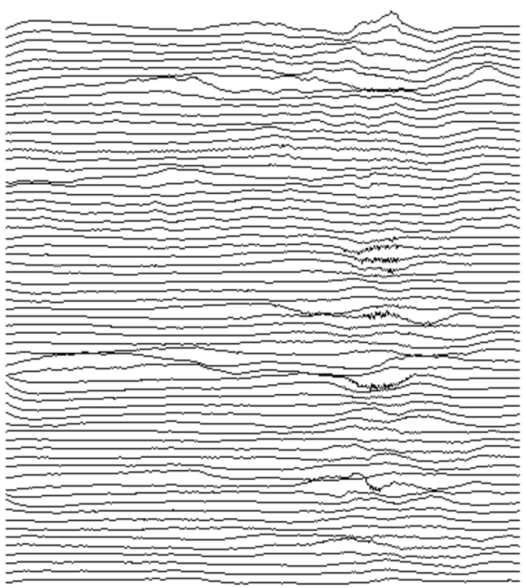

eeg_post.mat