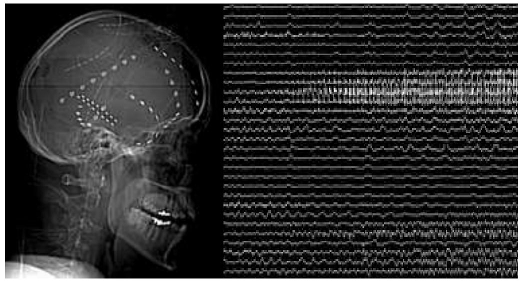
BENESCO Lecture Series on
Sleep, Epilepsy, Consciousness and Cognition

Bern, March 16th 2016

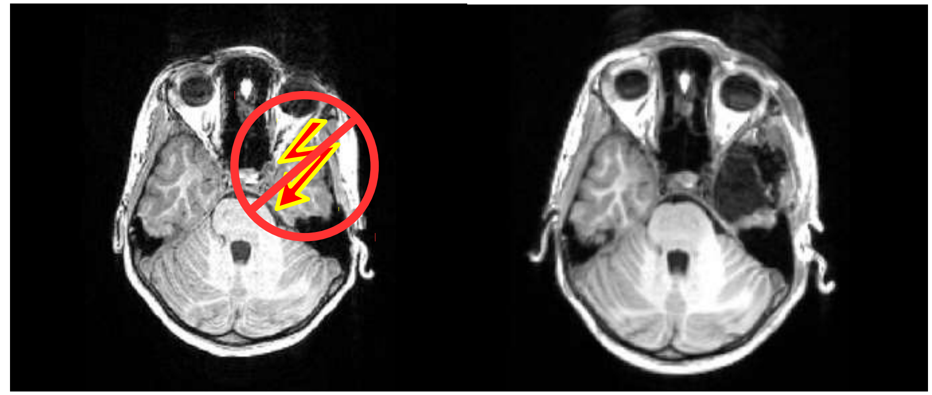


Correlation Analysis of Multivariate Time Series

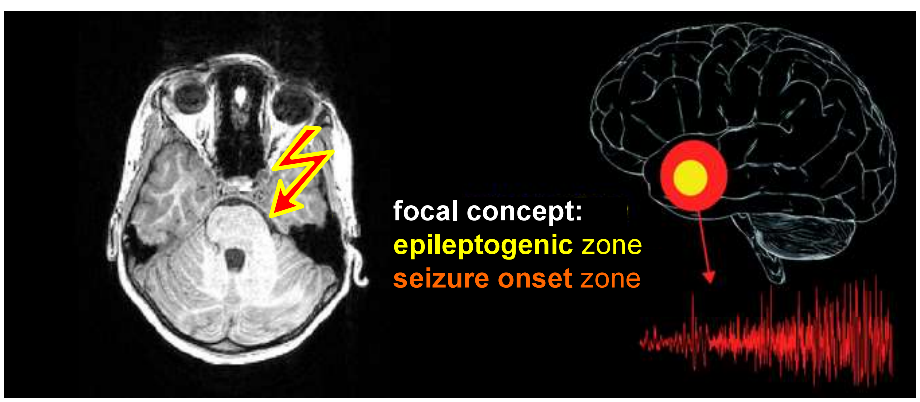
INSELSPITAL
UNIVERSITÄTSSPITAL BERN
HOPITAL UNIVERSITAIRE DE BERNE
BERN UNIVERSITY HOSPITAL

Christian Rummel
SCAN, University Institute of Diagnostic and Interventional Neuroradiology
University of Bern, Inselspital
christian.rummel@insel.ch

Diagnostics for Epilepsy Surgery

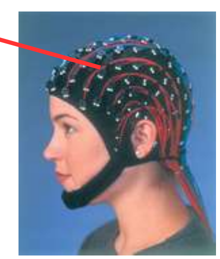


Diagnostics for Epilepsy Surgery

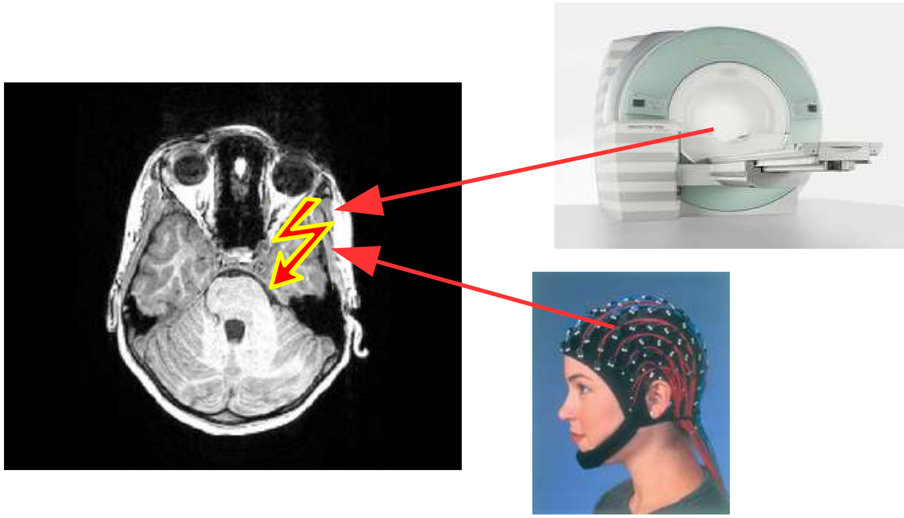


Rosenow & Lüders, Brain 124 (2001)

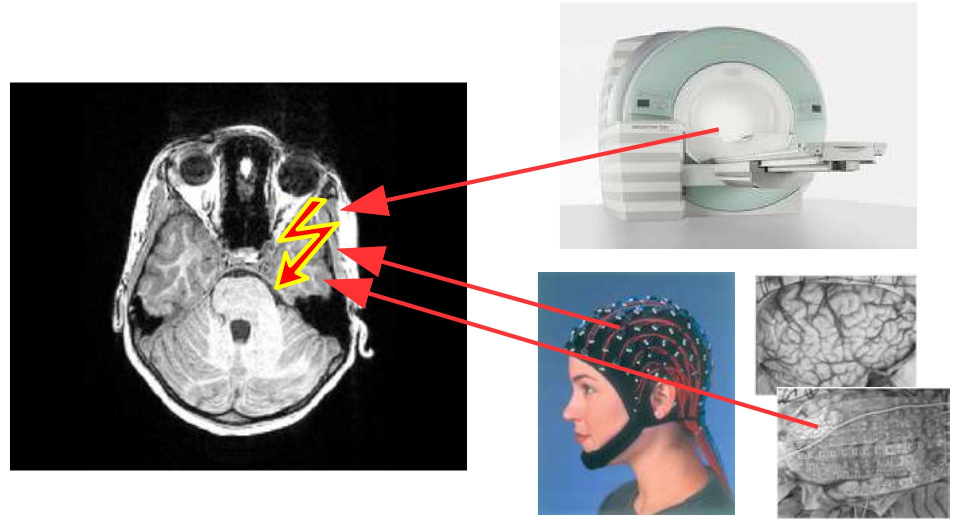
Diagnostics for Epilepsy Surgery



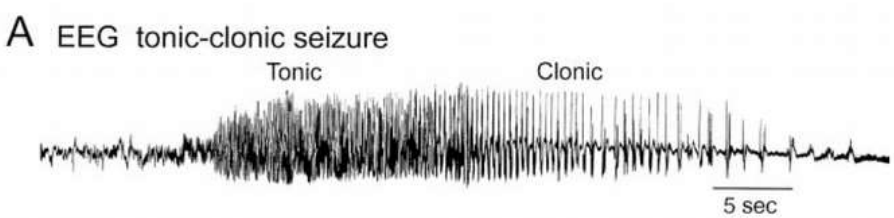
Diagnostics for Epilepsy Surgery

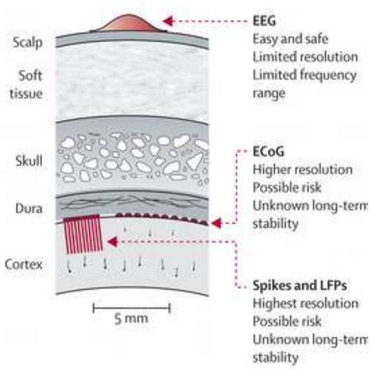


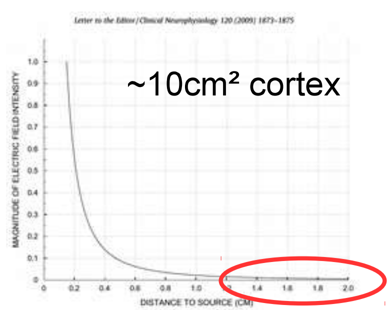
Diagnostics for Epilepsy Surgery



Hypersynchrony and Spatial Scales

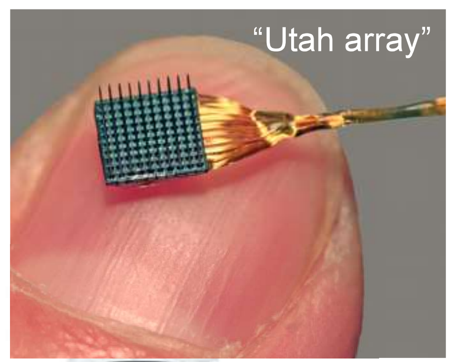


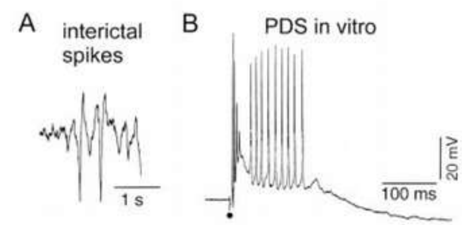


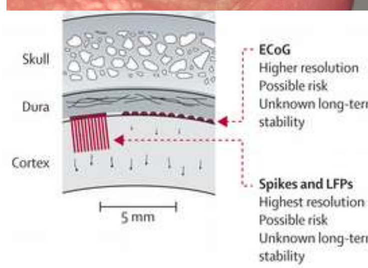


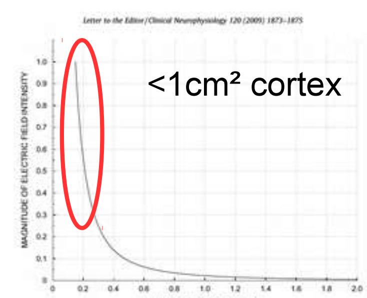
Zaveri et al., Clin. Neurophysiol. 120 (2009)

Hypersynchrony and Spatial Scales



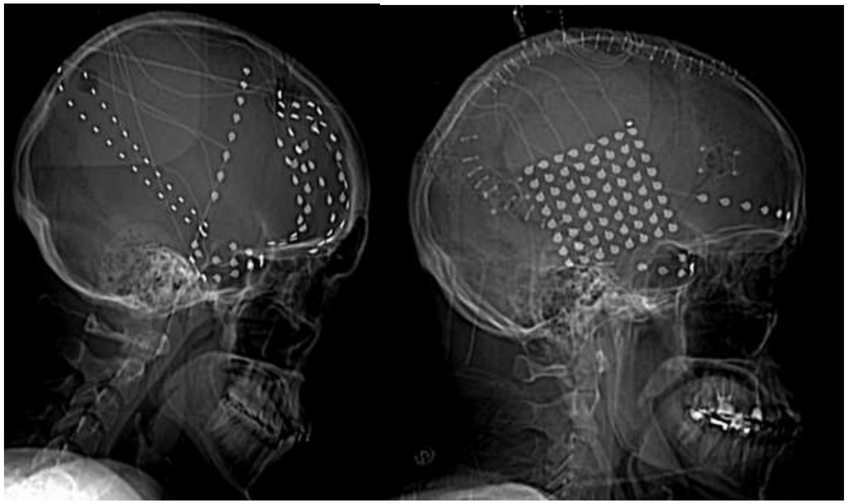




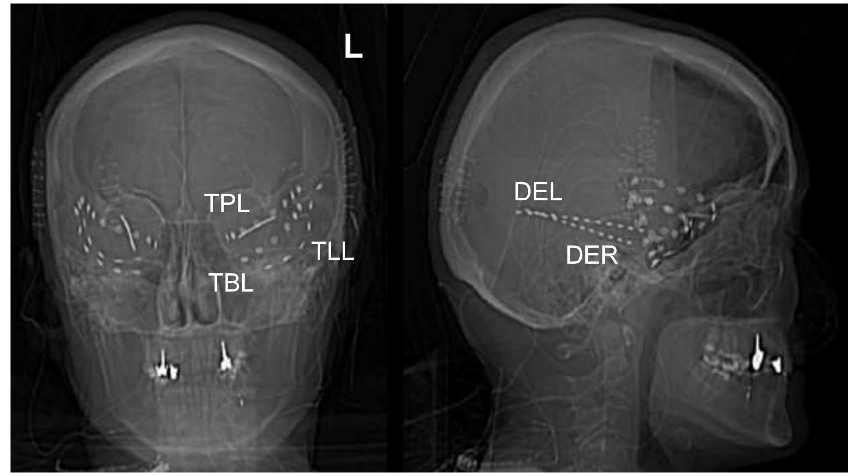


Zaveri et al., Clin. Neurophysiol. 120 (2009)

Hypersynchrony and Spatial Scales



Example EEG



Example EEG

```
[tme,EEG] =
display_EEG ('./data/','EEG_lecture.mat',128);
```

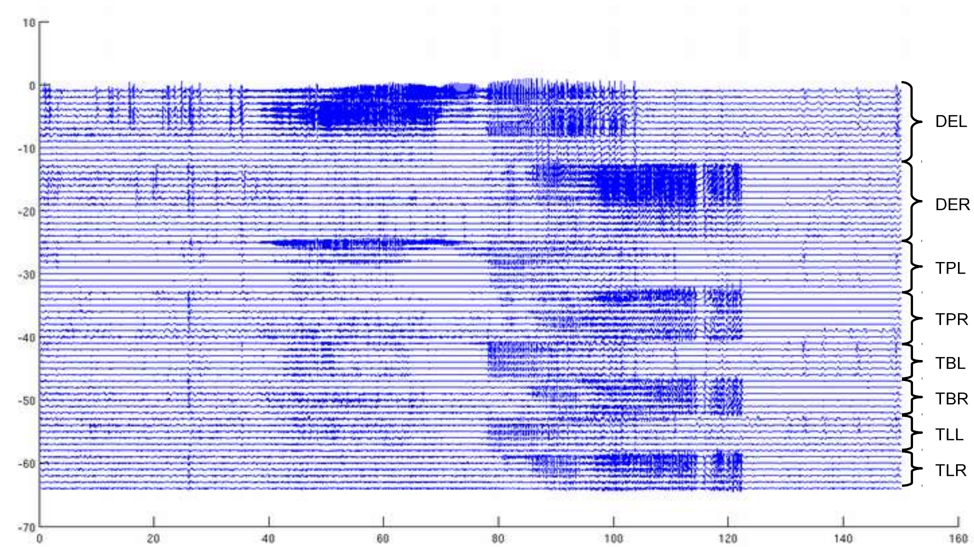
input:

'./data/'	path to data folder
'EEG_lecture.mat'	name of data file
128	sampling rate in Hertz

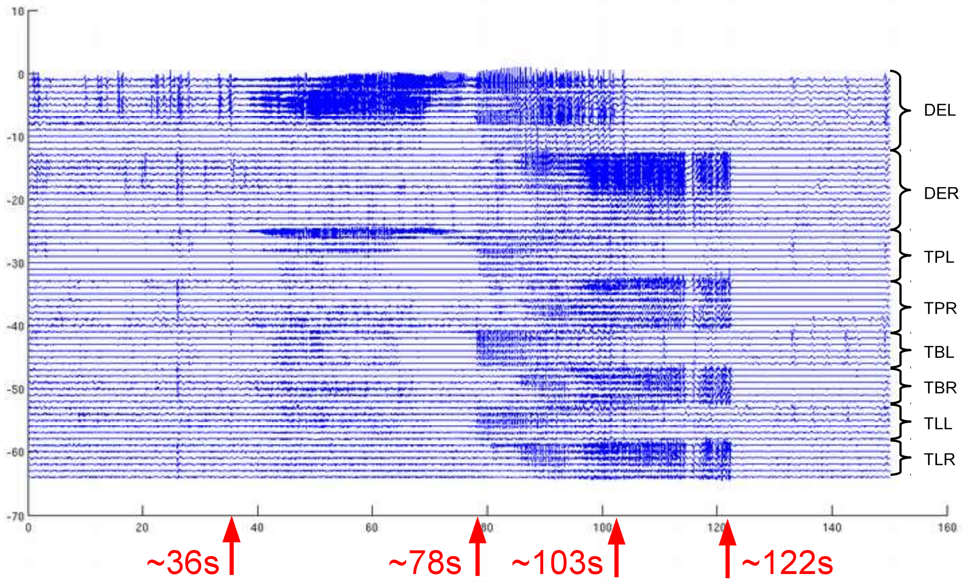
output:

tme	column vector containing sample times
EEG	matrix containing EEG as columns

Example EEG



Example EEG



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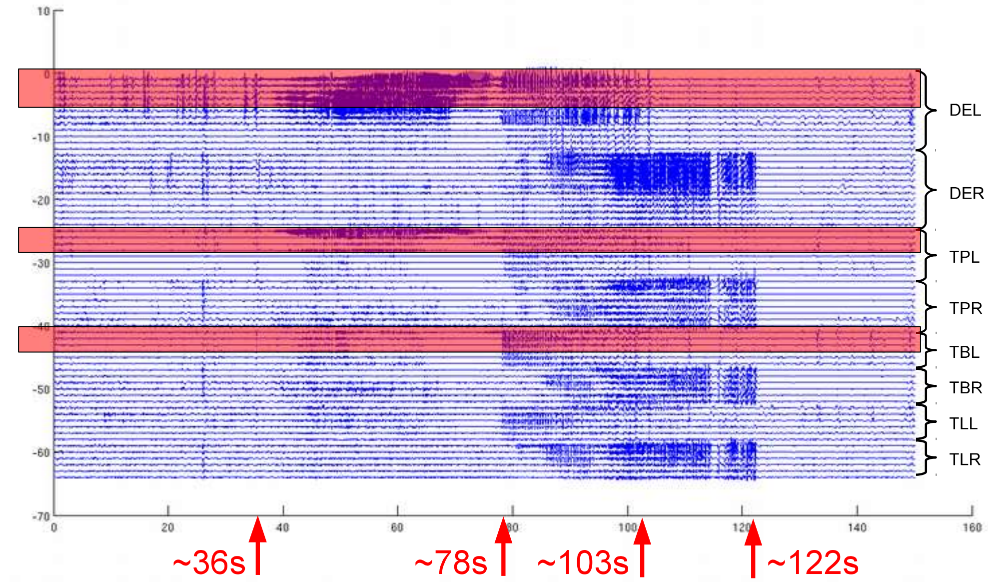
Example EEG



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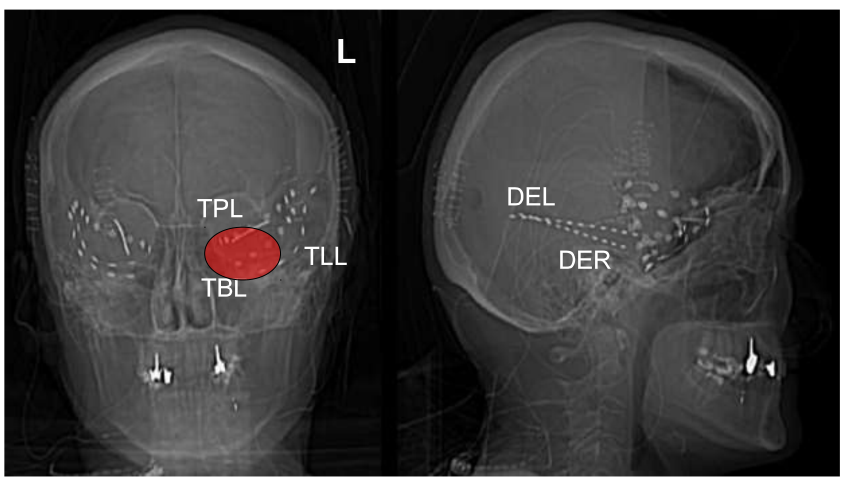
Example EEG



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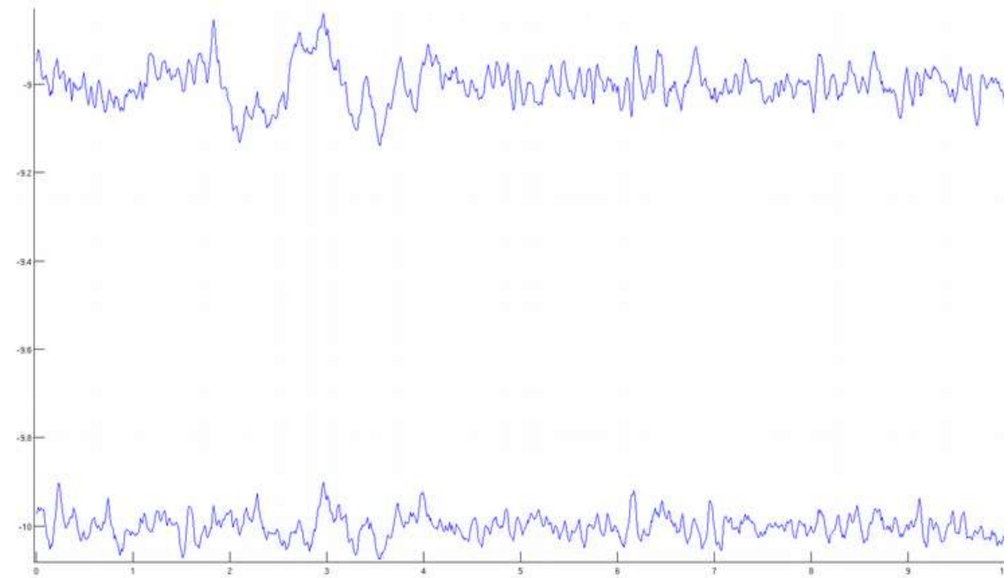
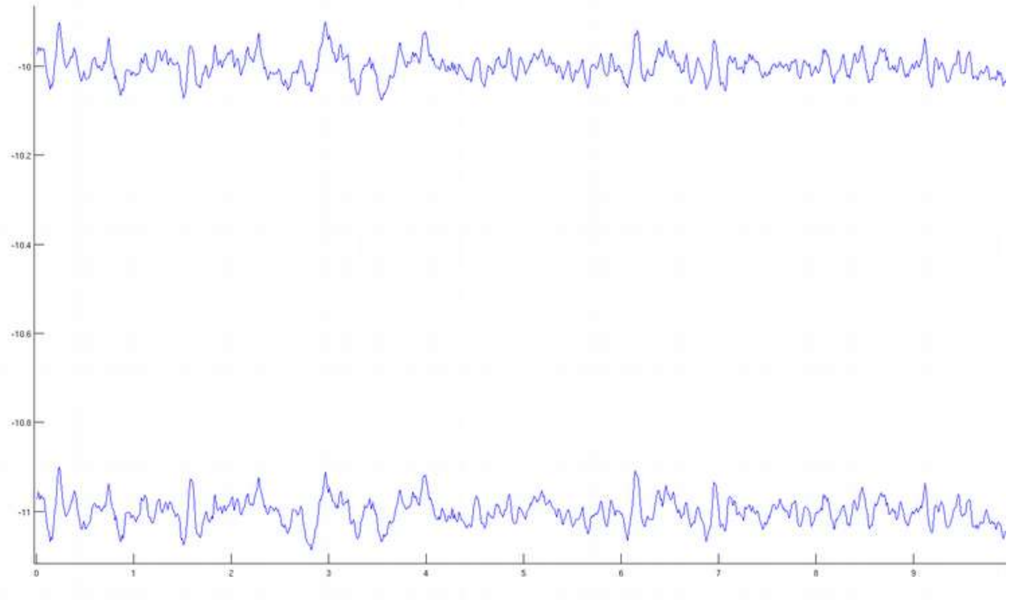
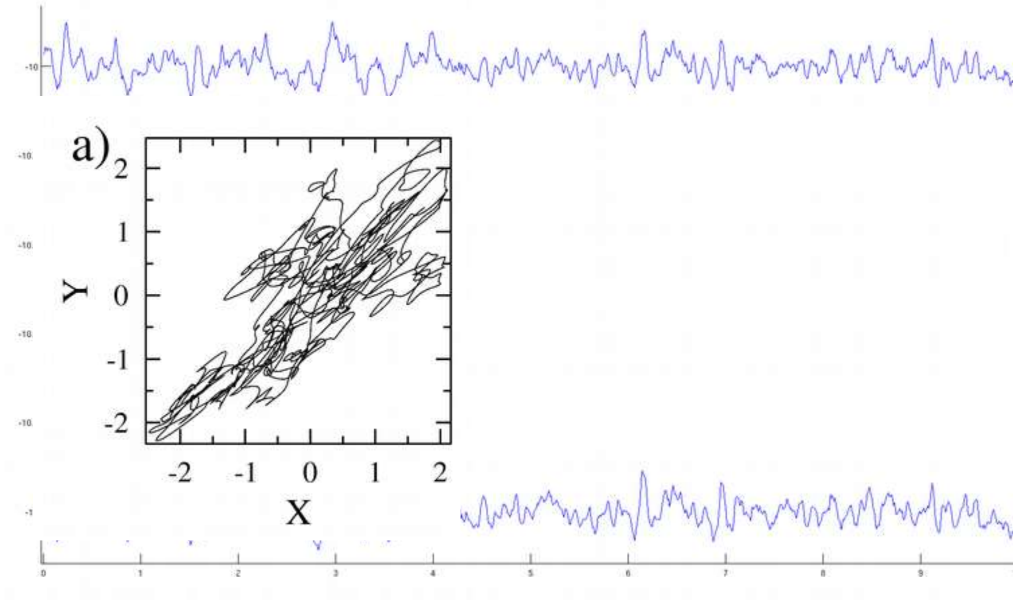
Pearson's Correlation Coefficient



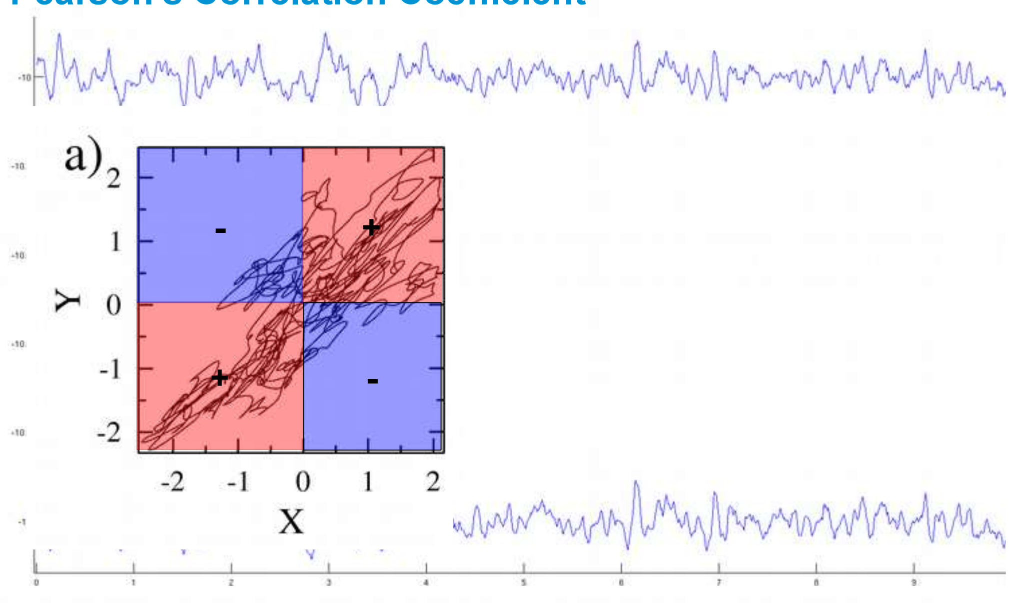
Pearson's Correlation Coefficient



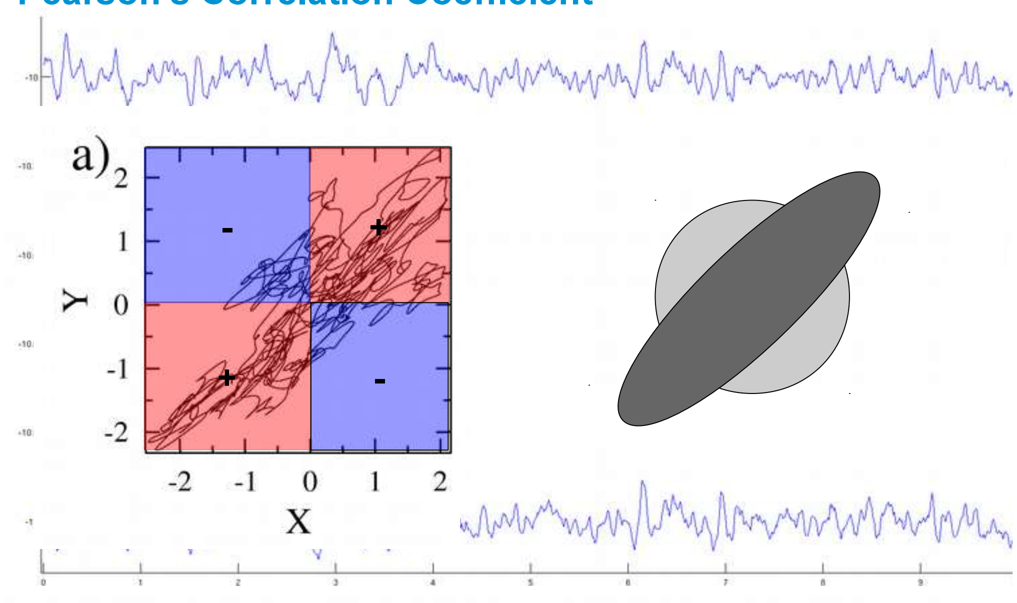
Pearson's Correlation Coefficient



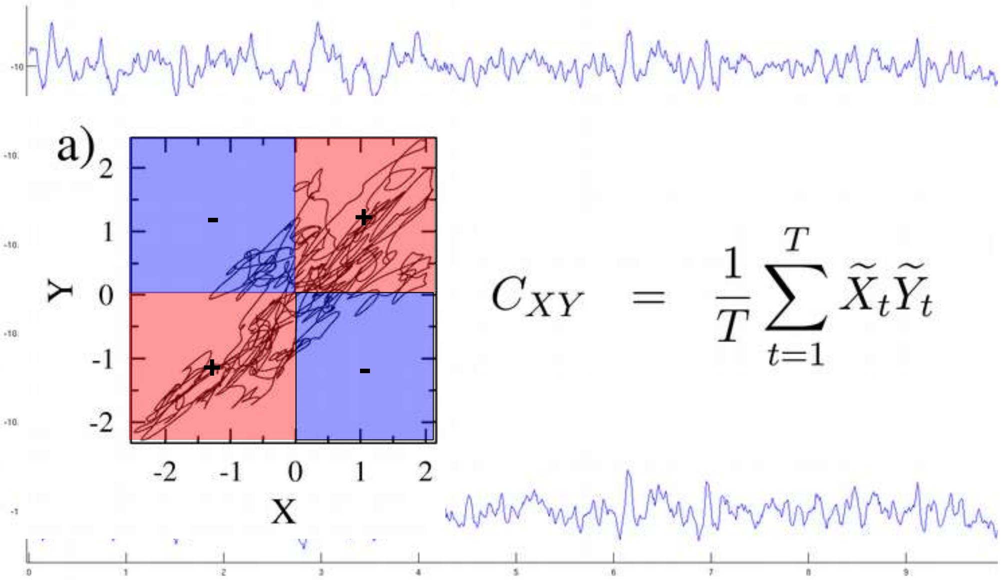
Pearson's Correlation Coefficient



Pearson's Correlation Coefficient



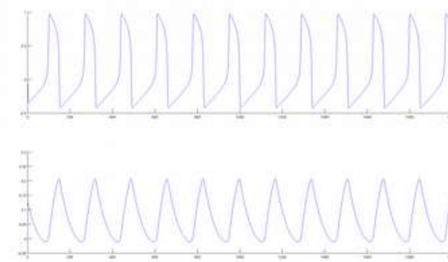
Pearson's Correlation Coefficient

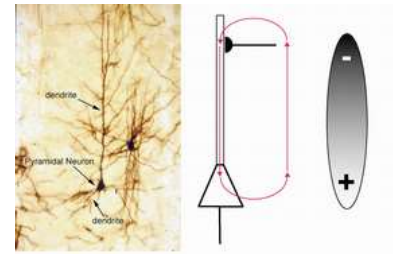


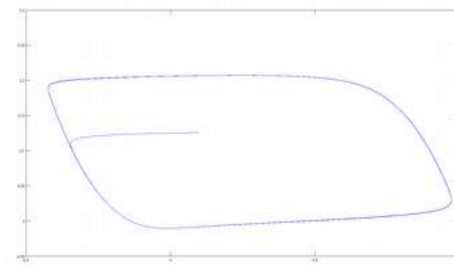
$$C_{XY} = \frac{1}{T}\sum_{t=1}^{T} \widetilde{X}_t \widetilde{Y}_t$$

FitzHugh-Nagumo Model

$$\frac{dv}{dt} = v(v-\alpha)(1-v) - w + I$$
$$\frac{dw}{dt} = \varepsilon(v - \gamma w).$$







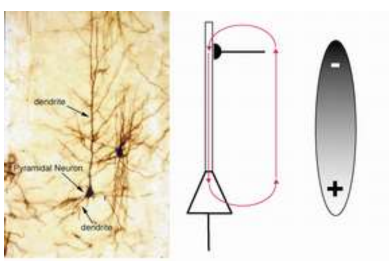
see BENESCO Lecture on 24th Nov., 2017

FitzHugh, Biophys J 1 (1961)

FitzHugh-Nagumo Model

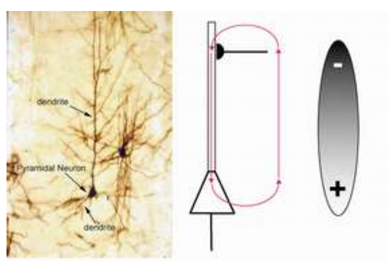
system 1: v_1, w_1

$$\frac{dv}{dt} = v(v-\alpha)(1-v) - w + I$$
$$\frac{dw}{dt} = \varepsilon(v - \gamma w).$$



system 2: v_2, w_2

$$\frac{dv}{dt} = v(v-\alpha)(1-v) - w + I$$
$$\frac{dw}{dt} = \varepsilon(v - \gamma w).$$

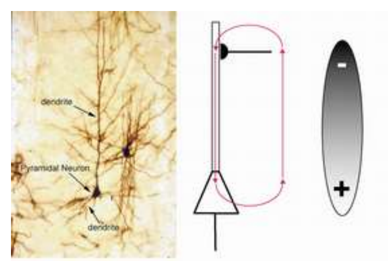


FitzHugh, Biophys J 1 (1961)

FitzHugh-Nagumo Model

system 1: v_1, w_1

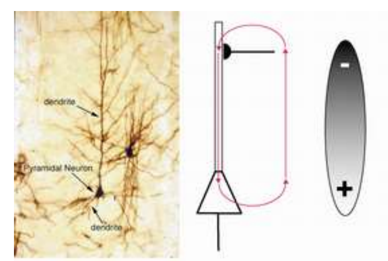
$$\frac{dv}{dt} = v(v-\alpha)(1-v) - w + I$$
$$\frac{dw}{dt} = \varepsilon(v - \gamma w).$$



system 2: v_2, w_2

coupling term: $+ c\,(v_1 - v_2)$

$$\frac{dv}{dt} = v(v-\alpha)(1-v) - w + I$$
$$\frac{dw}{dt} = \varepsilon(v - \gamma w).$$



FitzHugh, Biophys J 1 (1961)

FitzHugh-Nagumo Model

```
FitzHughNagumo2_coupling;
```

input:

–

output:

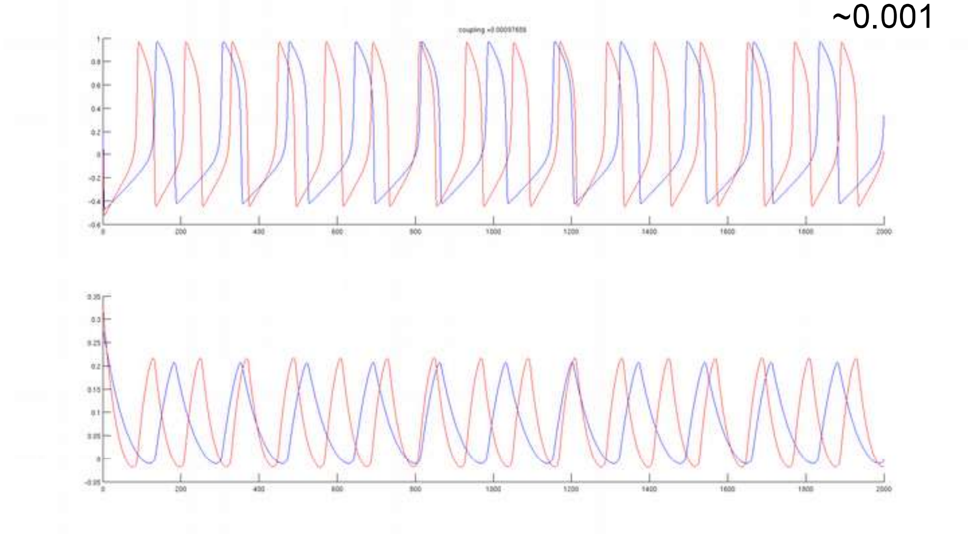
–

FitzHugh-Nagumo Model

coupling

1 → 2

~0.001

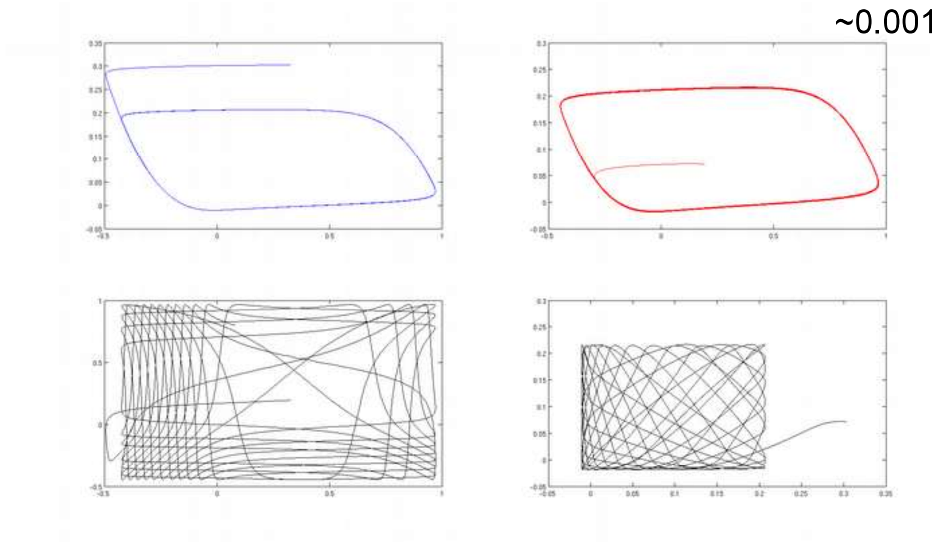


FitzHugh-Nagumo Model

coupling

1 → 2

~0.001

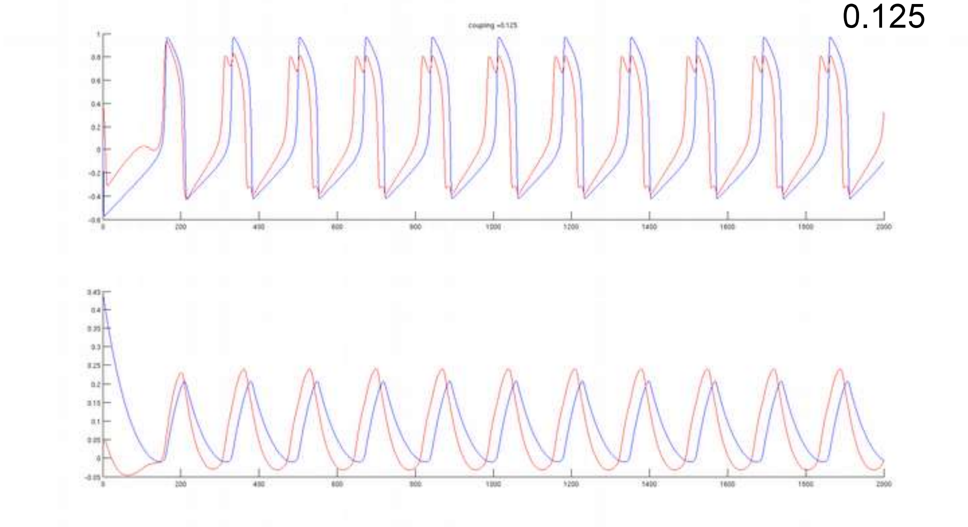


FitzHugh-Nagumo Model

coupling

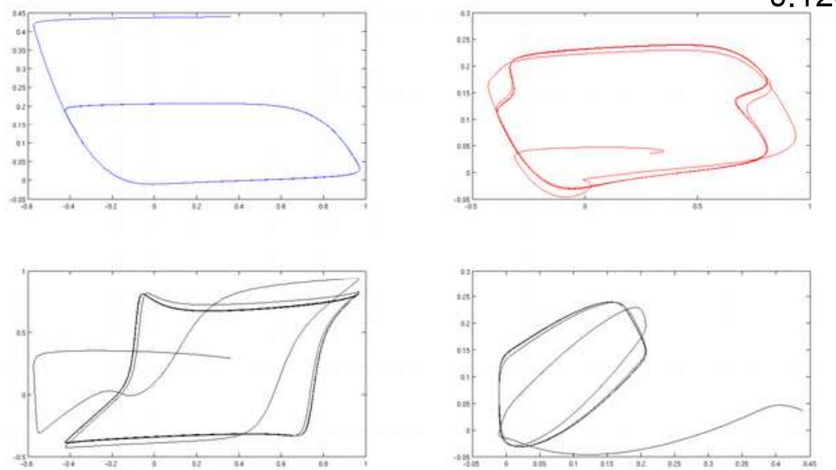
1 → 2

0.125



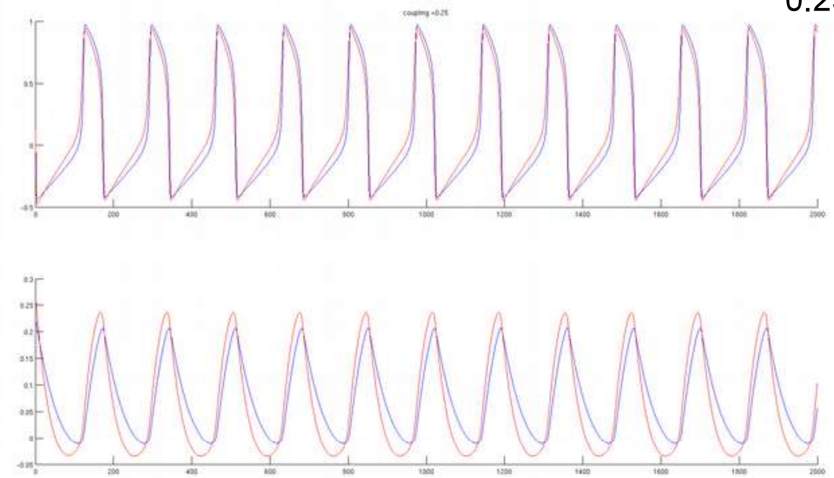
FitzHugh-Nagumo Model

coupling
1 → 2
0.125



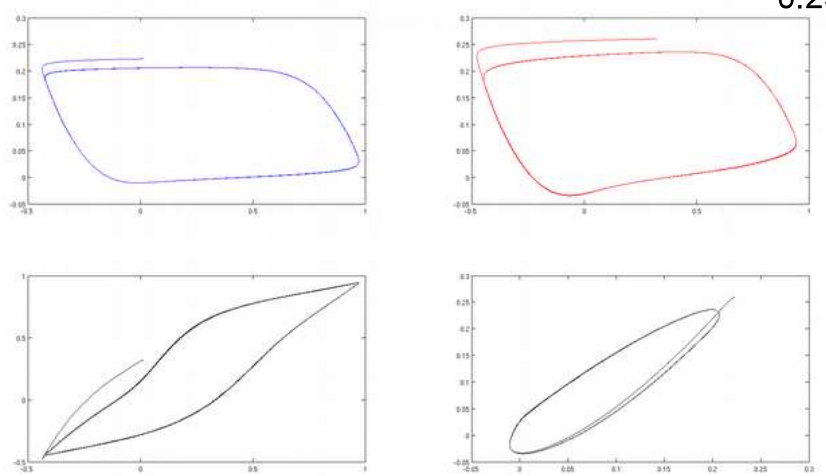
FitzHugh-Nagumo Model

coupling
1 → 2
0.25

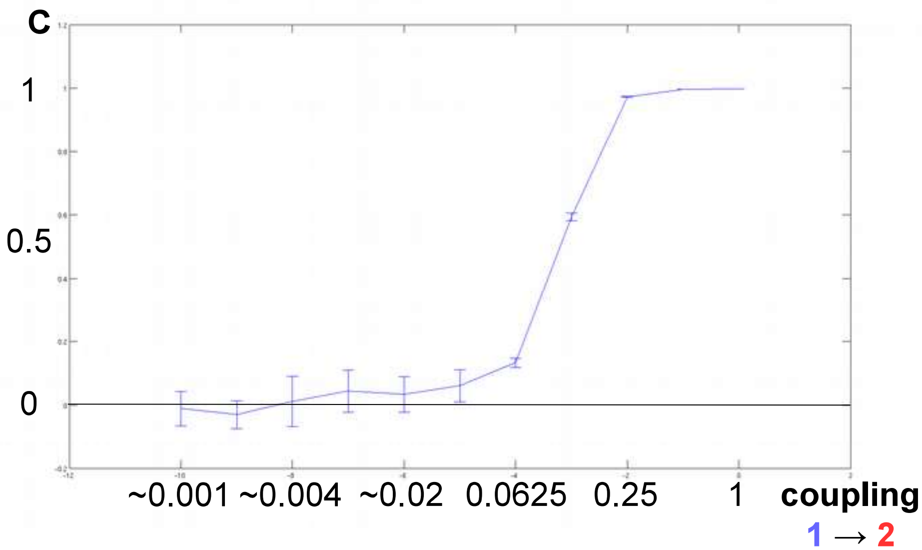


FitzHugh-Nagumo Model

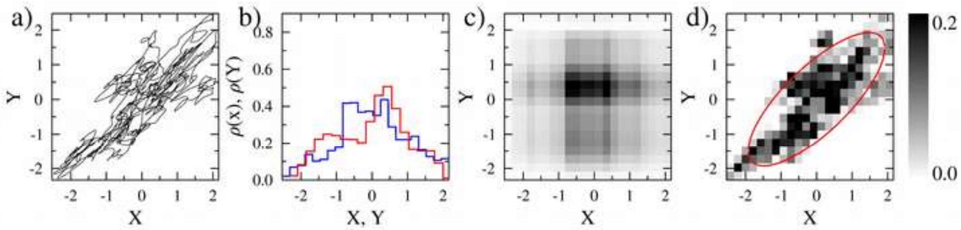
coupling
1 → 2
0.25



FitzHugh-Nagumo Model

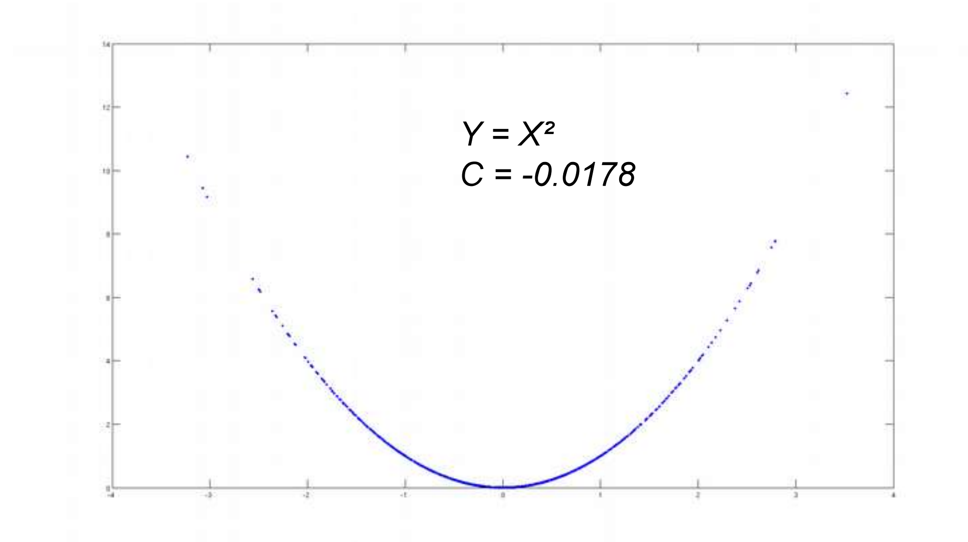


Other Interrelation Measures

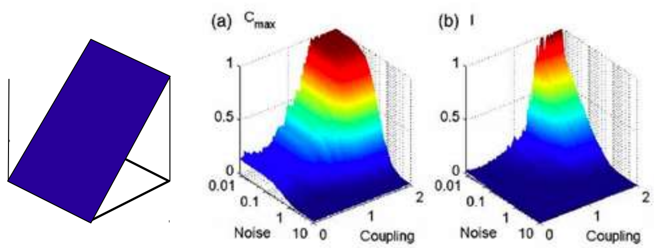


cross-correlation
vs.
mutual information

Other Interrelation Measures

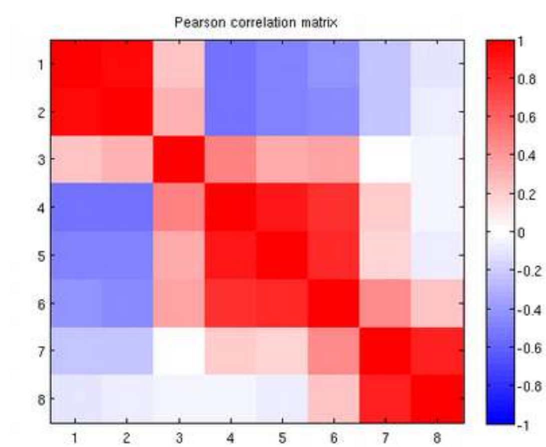


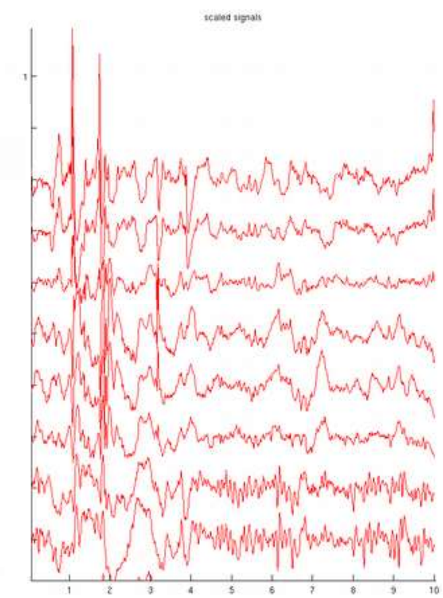
Other Interrelation Measures



Kreuz et al., Physica D225 (2007)

Pearson's Correlation Matrix





Pearson's Correlation Matrix

mathematical properties:

matrix form $$C_{ij} = \frac{1}{T}\sum_{t=1}^{T} \frac{X_i(t) - \langle X_i \rangle}{\sigma_i} \frac{X_j(t) - \langle X_j \rangle}{\sigma_j} \qquad \mathbf{C} = \frac{1}{T}\tilde{\mathbf{X}}\tilde{\mathbf{X}}^t$$

values on the diagonal $$C_{ii}(t) \equiv 1$$

symmetry $$C_{ij}(t) = C_{ji}(t)$$

range of values $$-1 \le C_{ij}(t) \le 1$$

positive semi-definite $$\lambda_i \ge 0$$

trace $$\mathrm{Tr}\,\mathbf{C} = \sum_{i=1}^{M} \lambda_i = \sum_{i=1}^{M} C_{ii} = M$$

Pearson's Correlation Matrix

```
EEG_Corr (tme,EEG, 640,640, 1);
```

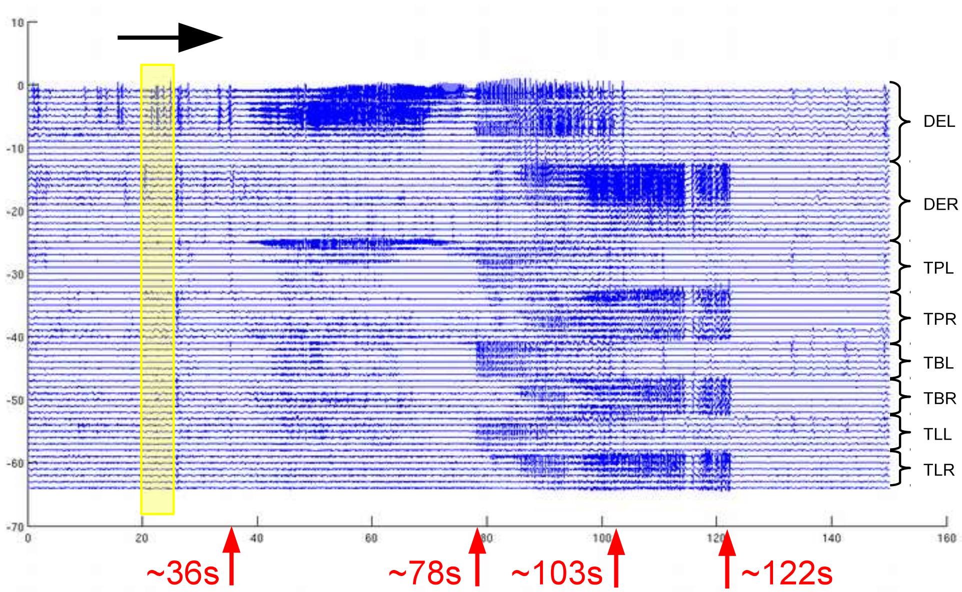
input:

`tme`	column vector containing sample times
`EEG`	matrix containing EEG as columns
`640`	number of samples used for correlation matrix
`640`	number of samples used for displacement
`1`	flag for display of intermediate results

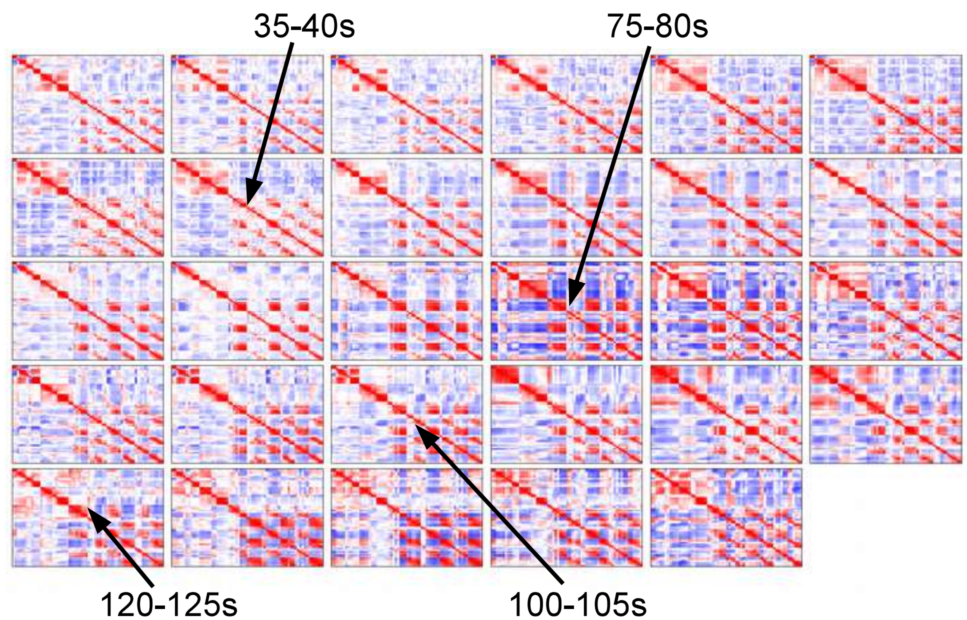
output:

–

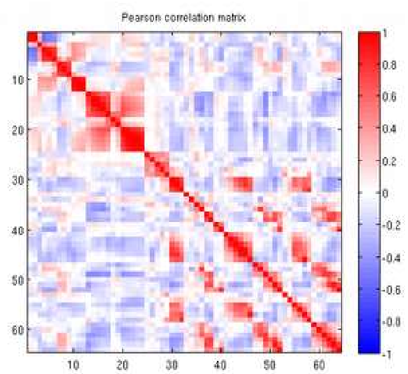
Pearson's Correlation Matrix



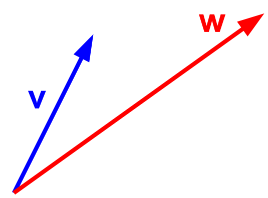
Pearson's Correlation Matrix



Eigenvalues and Eigenvectors

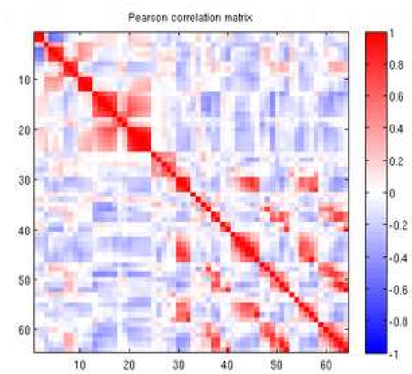


$$\mathbf{C}\,\mathbf{v} = \quad \mathbf{w}$$



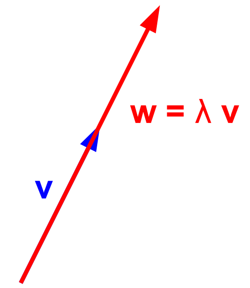
see BENESCO Lecture on 29[th] Sept., 2017

Eigenvalues and Eigenvectors



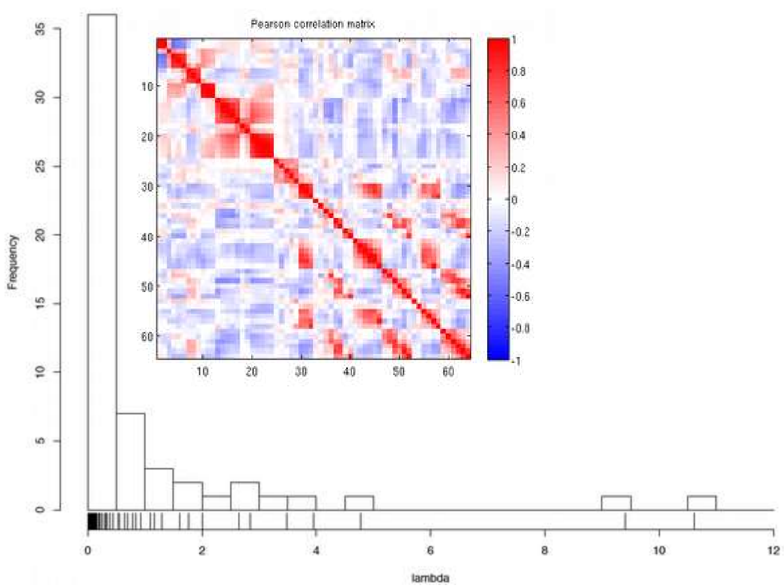
$$\mathbf{C}\,\mathbf{v} = \quad \mathbf{w}$$

$$\mathbf{C}\,\mathbf{v} = \lambda\,\mathbf{v}$$

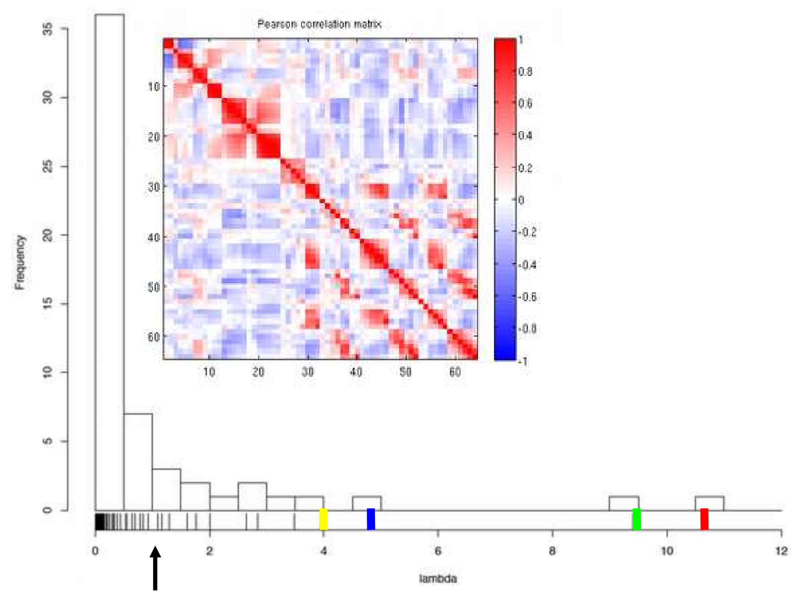


see BENESCO Lecture on 29[th] Sept., 2017

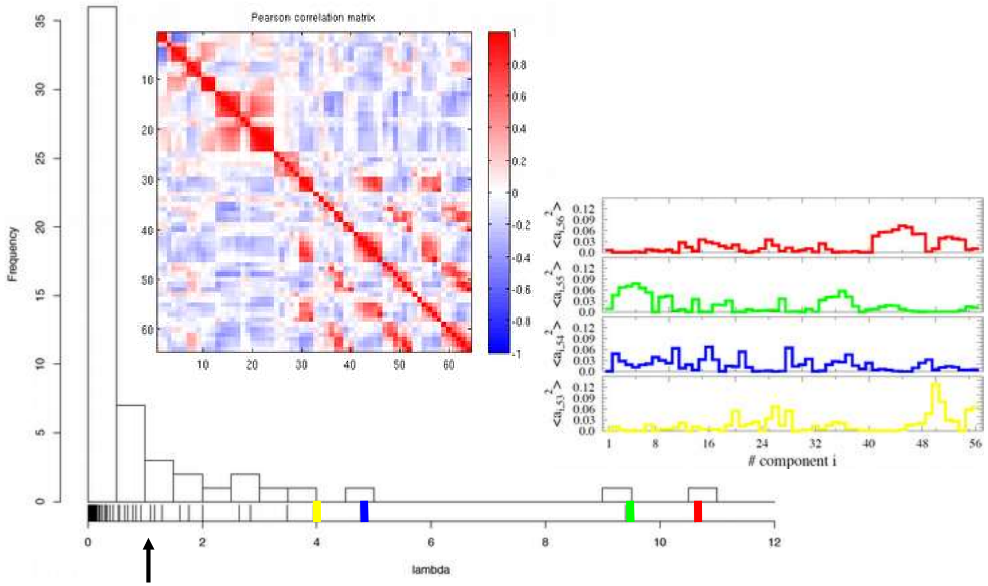
Eigenvalues and Eigenvectors



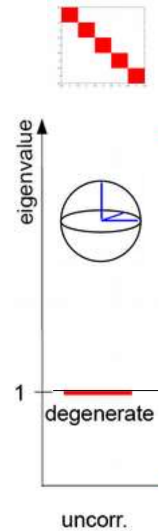
Eigenvalues and Eigenvectors



Eigenvalues and Eigenvectors



Eigenvalues and Eigenvectors

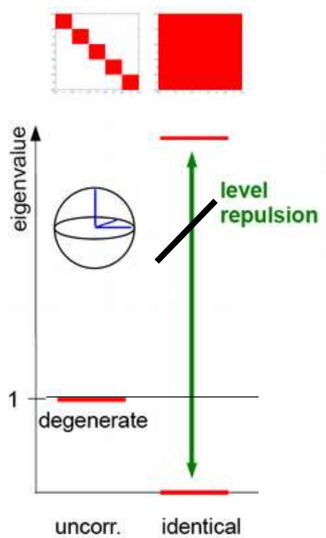


$$\sum_{i=1}^{M} \lambda_i = M$$

idealized

real world

Eigenvalues and Eigenvectors

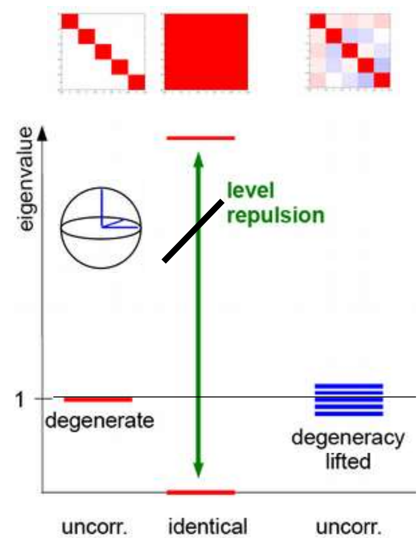


$$\sum_{i=1}^{M} \lambda_i = M$$

idealized

real world

Eigenvalues and Eigenvectors

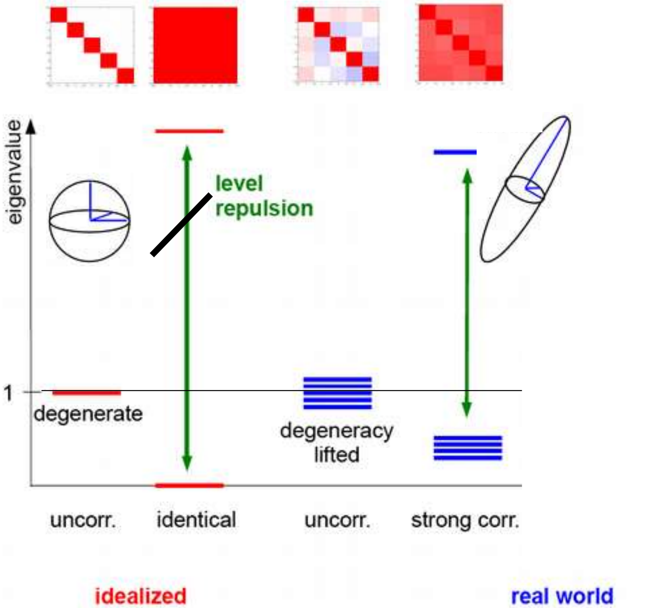


$$\sum_{i=1}^{M} \lambda_i = M$$

idealized

real world

Eigenvalues and Eigenvectors



$$\sum_{i=1}^{M} \lambda_i = M$$

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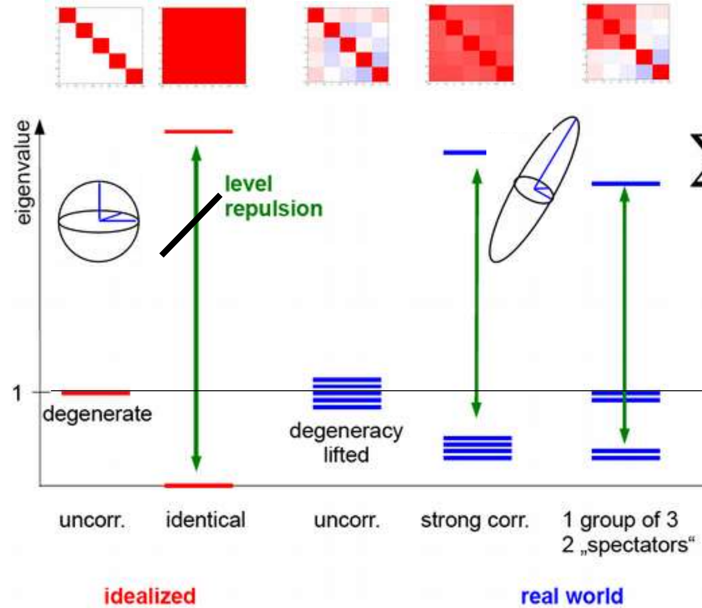
Eigenvalues and Eigenvectors



$$\sum_{i=1}^{M} \lambda_i = M$$

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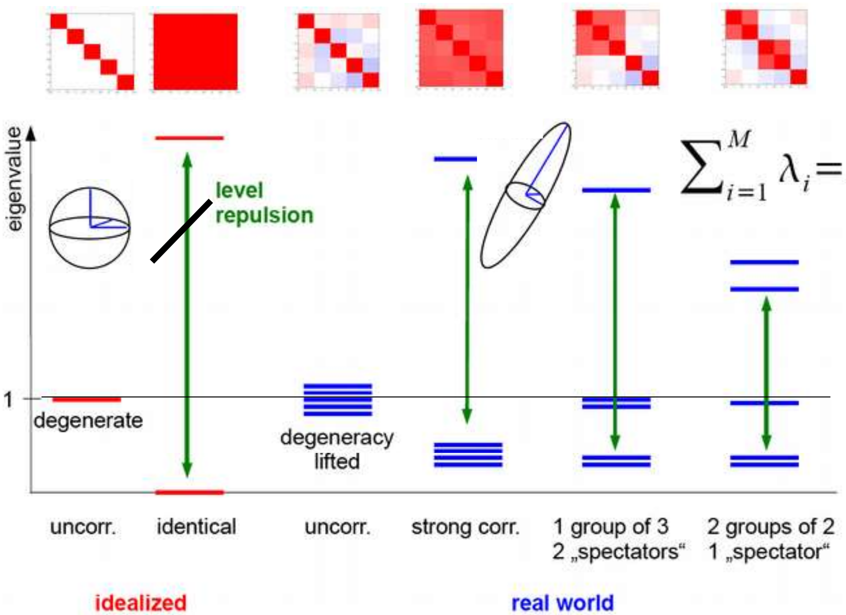
Eigenvalues and Eigenvectors



$$\sum_{i=1}^{M} \lambda_i = M$$

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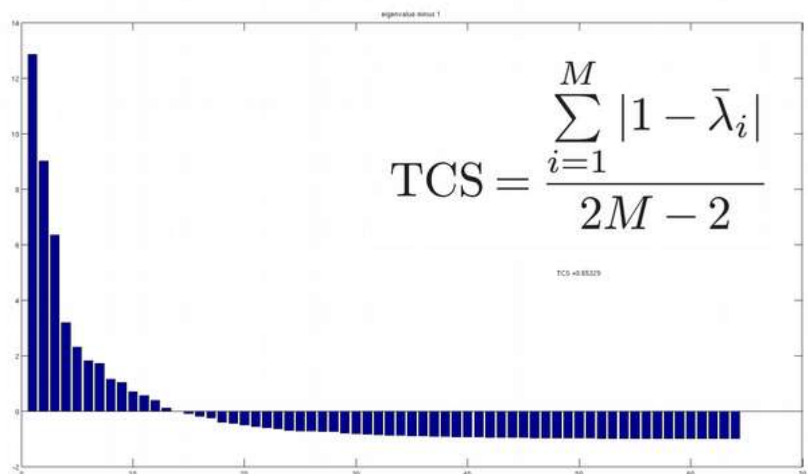
Eigenvalues and Eigenvectors

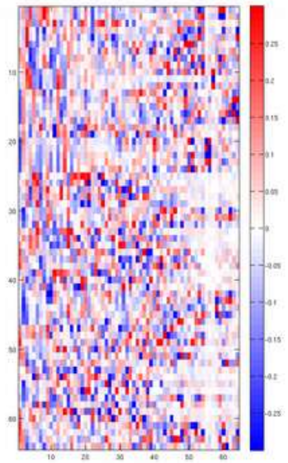


$$\mathrm{TCS} = \frac{\sum_{i=1}^{M} |1 - \bar{\lambda}_i|}{2M - 2}$$

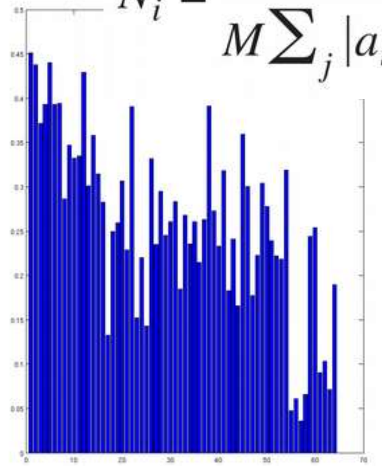
“total correlation strength”

Müller et al., Europhys. Lett. 84 (2008)

Eigenvalues and Eigenvectors



$$N_i^p = \frac{1}{M \sum_j |a_{ij}|^4}$$



"collectivity",
"number of principal components"

Müller et al., Phys. Rev. E 71 (2005)

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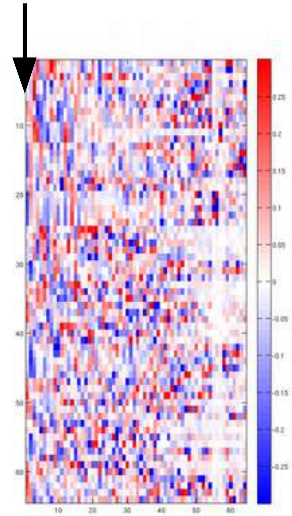
Eigenvalues and Eigenvectors



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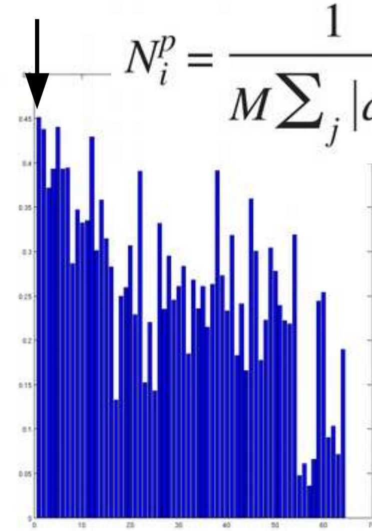

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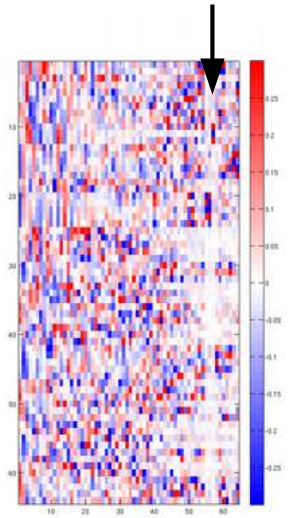

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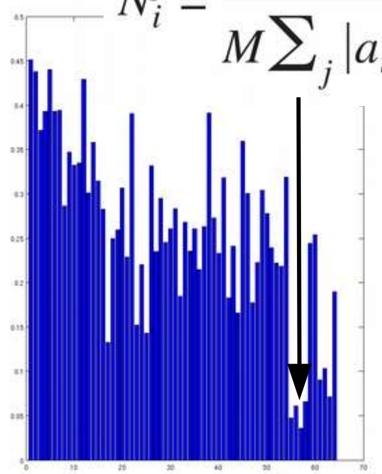

"collectivity",
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Müller et al., Phys. Rev. E 71 (2005)

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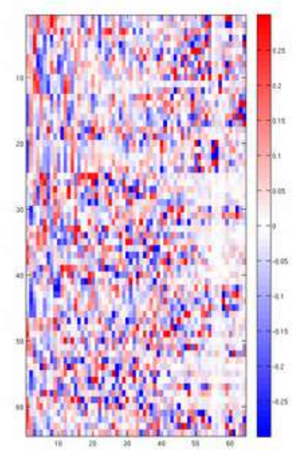
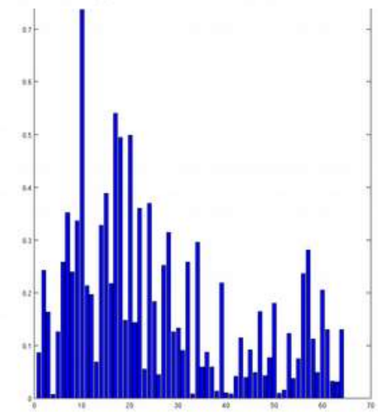
Eigenvalues and Eigenvectors

$$S_i = \left| \sum_j \operatorname{sgn}(a_{ij}) |a_{ij}|^2 \right|$$





"symmetry"

Müller et al., Phys. Rev. E 71 (2005)

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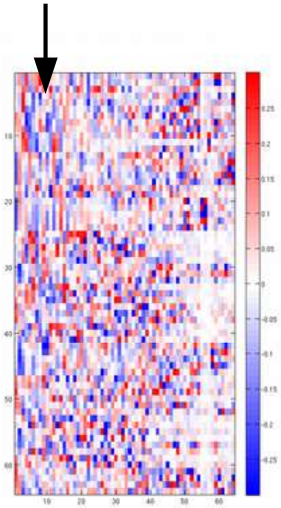

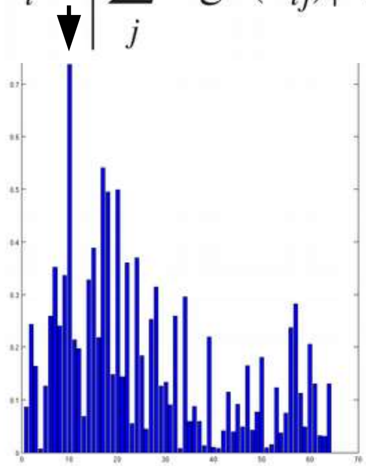

"symmetry"

Müller et al., Phys. Rev. E 71 (2005)

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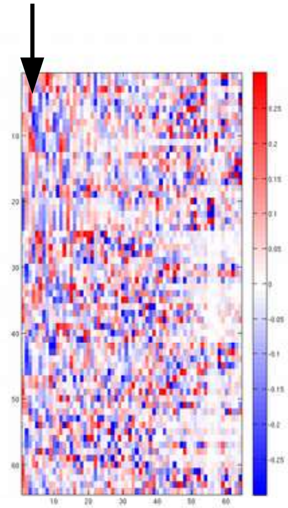

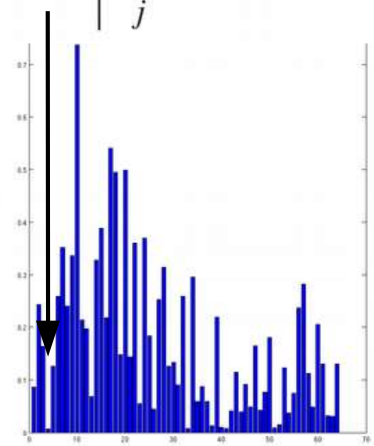

"symmetry"

Müller et al., Phys. Rev. E 71 (2005)

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Eigenvalues and Eigenvectors

```
eig_val_vec (tme,EEG, 640,128, 1);
```

input:

`tme`	column vector containing sample times
`EEG`	matrix containing EEG as columns
`640`	number of samples for correlation matrix
`128`	number of samples for displacement
`1`	flag for display of intermediate results

output:

–

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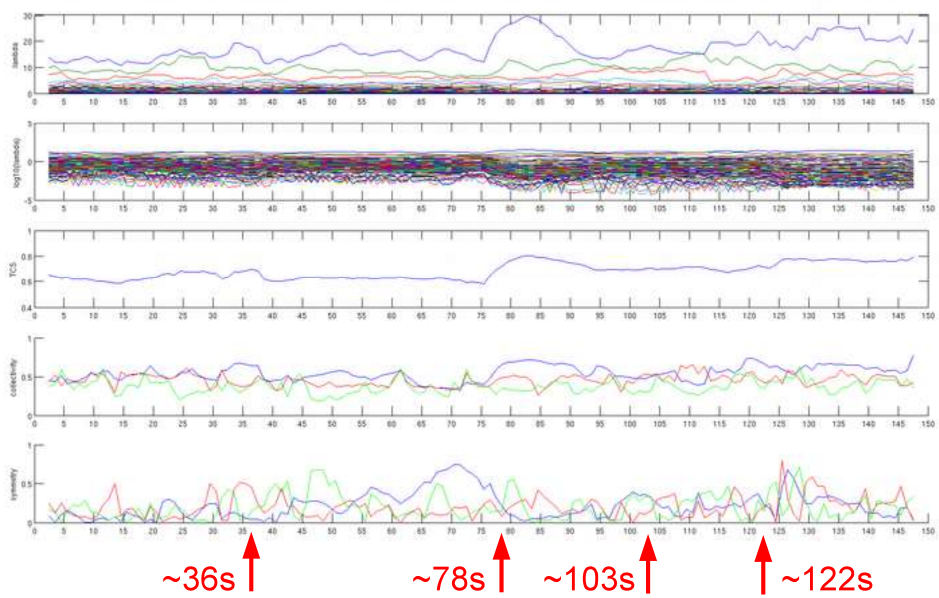
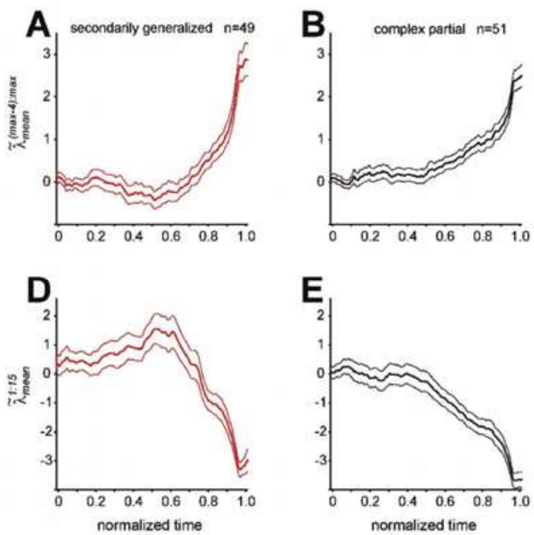
Eigenvalues and Eigenvectors



Eigenvalues: correlation dynamics of seizures



Schindler et al., Brain 130 (2007)

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Literature suggestions

- Müller et al. (2005), Phys. Rev. E71, 046116.
- Rummel et al. (2013), Neuroinformatics 11, 159-173.
- Schindler et al. (2007), Brain 131, 65-77.

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Homework suggestions

Use the Matlab functions `EEG_Corr.m` and `eig_val_vec.m` to **analyze** the data set `EEG_homework.mat`:

1) When and where do you think the **seizure starts**?
2) When do you think the **seizure terminates**?
3) Repeat analysis for the first temporal **derivative** `diff(EEG,1)`.
4) **Which seizure** of the Supplementary Material of Rummel et al. (2013) is it?