Nonlinear signal analysis: An intuititive introduction without formulae

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Nonlinear signal analysis: An intuititive introduction without formulae

Literature

H Kantz, T Schreiber, Nonlinear time series analysis, 2nd ed. Cambridge University Press, Cambridge, 2004

&

RG Andrzejak, Nonlinear time series analysis in a nutshell. Osorio et al. (eds.) Epilepsy: The Intersection of Neurosciences, Biology, Mathematics, Engineering and Physics. PDF available at ntsa.upf.edu

Deterministic versus stochastic dynamics

A dynamical system is called deterministic if its temporal evolution is fully determined by its initial conditions. It can be defined, for example, by a set of differential equations.

For stochastic dynamics the temporal evolution is not unambiguously determined by rules such as differential equations. Rather the temporal evolution is governed by some random process.

Motivation

SIMULATION

- Visual inspection of exemplary signals
  - Classification as random or regular
  - Can we make this classification using the power spectrum?

Measuring the harmonic oscillator

SIMULATION

- Harmonic oscillator: a simple dynamical system
  - Temporal evolution of components
  - Signal
  - State space representation

Lorenz dynamics

SIMULATION

- Lorenz dynamics
  - Three components
  - State space representation

Reconstructing the state space

SIMULATION

- Harmonic oscillator
  - original signals and state space
  - delayed signals and reconstructed state space

Delay coordinates

SIMULATION

- We apply delay coordinates to Lorenz dynamics
  - influence of time delay τ

Distance matrices

SIMULATION

- Distance matrices
  - Original dynamics
  - Reconstructed dynamics
- Influence of time delay τ
- Influence of embedding dimension m

Trajectories of deterministic dynamics ...

SIMULATION

- Inspection of trajectories of deterministic dynamics
- Nearby traces of the trajectories

. . . do not intersect

For any given initial condition the future evolution of a deterministic dynamical system is fully determined.

$\Rightarrow$ Trajectories of deterministic dynamical systems cannot intersect. Nearby trajectory segments are aligned.

For any given initial condition the future evolution of a stochastic dynamics is not determined.

$\Rightarrow$ Trajectories of stochastic dynamics can intersect. Nearby trajectory segments point to different directions.

We quantify this criterion of distinction with the nonlinear prediction error.

Nonlinear prediction error

SIMULATION

- Nonlinear prediction error
  - Lorenz
  - Stochastic signals with different correlations
  - Noisy Lorenz
- Influence of time delay τ

Nonlinear prediction error - results

The nonlinear prediction error shows:

- Zero values for periodic dynamics
- Low values for deterministic dynamics
- High values for white noise
- Intermediate values for dynamics that are
  - noisy deterministic
  - noisy periodic
  - correlated stochastic
- Strong dependence on time delay τ

Nonlinear prediction error - Power and limitations

The nonlinear prediction error is sensitive to deterministic structure without being fully specific:

- Strength: detects deterministic structure
- Strength: distinguishes deterministic dynamics and white noise
- Limitation: overlapping results for noisy deterministic and correlated stochastic signals
- Limitation: strong influence of linear correlations

We need a further ingredient:

$\Rightarrow$ The concept of surrogates

The concept of surrogates

- Surrogate signals: very powerful and versatile tool in signal analysis.
- Allow to test a variety of well-defined null hypotheses about the dynamics underlying a signal.
- Essential to judge results of nonlinear signal analysis measures.
- We will introduce them empirically using the example of phase randomized surrogates.

Phase randomized surrogates - algorithm

Algorithm for the generation of phase randomized surrogates

- Compute the Fourier transform of the signal
- Randomize all phase angles of the Fourier coefficients
- Compute the inverse Fourier transform of the randomized Fourier coefficients

true positive & true negative null hypothesis rejections

SIMULATION

- Nonlinear prediction error for original signal versus phase randomized surrogates
  - Lorenz dynamics
  - Noisy Lorenz dynamics
  - Stochastic signals
    - Influence of autocorrelation

Outcome

Nonlinear prediction error for original signals and phase randomized surrogates

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coincides for

- Stochastic signals (regardless of autocorrelation)

$\Rightarrow$ null hypothesis not rejected

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does not coincide for

- Lorenz dynamics
- Noisy Lorenz dynamics

$\Rightarrow$ null hypothesis rejected

Interpretation

Combination of the nonlinear prediction error with phase randomized surrogates $\Rightarrow$ more specific for deterministic structure than the nonlinear prediction error alone.

If null hypothesis is rejected $\Rightarrow$ some indication that dynamics is non-random.

But careful:

- If the null hypothesis is rejected $\Rightarrow$ no proof that dynamics is indeed deterministic.
  - Likewise dynamics could be non-stationary, nonlinear stochastic, non-Gaussian, etc.
- If the null hypothesis is not rejected $\Rightarrow$ no proof that it is indeed correct.
  - Likewise test statistics might not be sensitive enough to detect deterministic structure, nonlinearity, non-stationarity, non Gaussianity, etc.

not a false positive null hypothesis rejection

SIMULATION

- Nonlinear prediction error for original signal versus phase randomized surrogates
  - Non-stationary signals

So what?

Why should the Bern Network for Epilepsy, Sleep and Consciousness care about these results?